Imaging Theory for X-Ray Pixel Detectors

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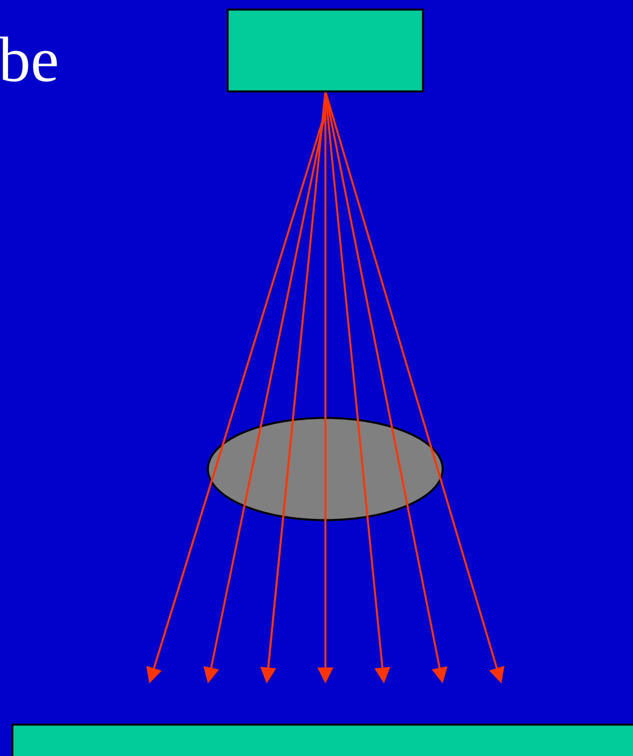
IWORID 2005

X-Ray Imaging Elements

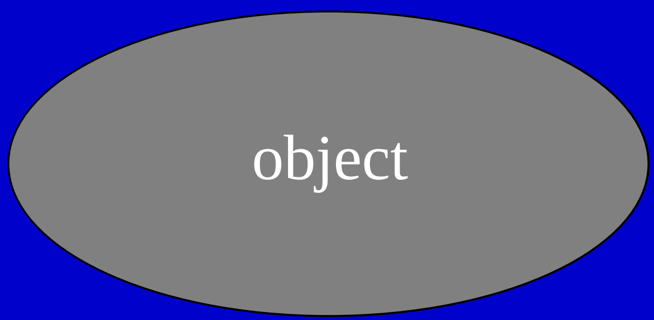


$$I_o(E)$$

$$\int_0^L \mu\ (E, \vec{r})\ dl$$

$$I(E, x, y)$$

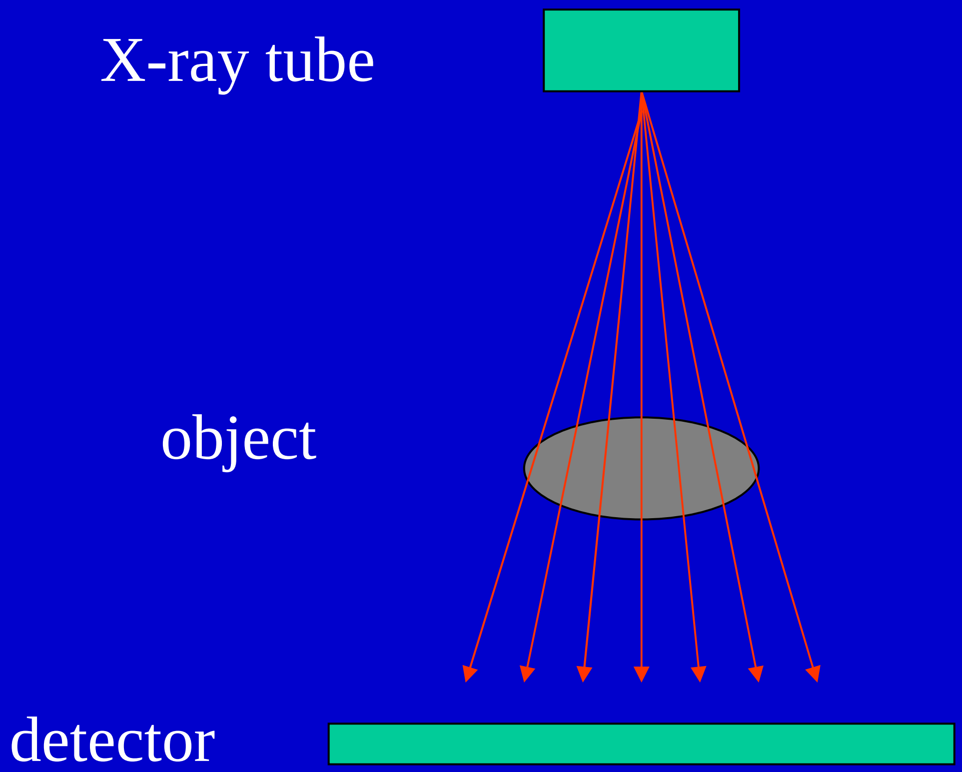
X-Ray Image



$$\mu(E, \vec{r})$$

attenuation coefficient

X-Ray Imaging Elements



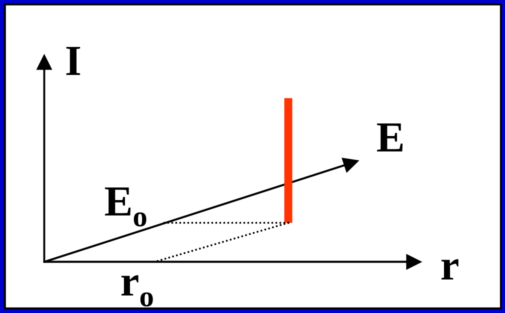
$$I_o(E)$$

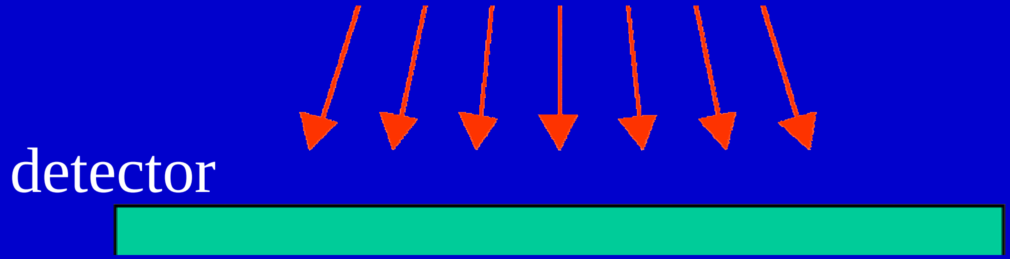
$$\int_0^L \mu\ (E, \vec{r})\ dl$$

$$I(E, x, y)$$

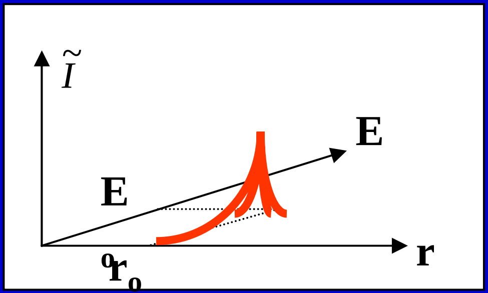
Detector Transfer Function

$$I(E_o, x_o, y_o)$$





$$\tilde{I}(E, x, y)$$



Tools

A. Simulation calculations:

→ ROSI (Erlangen)

B. Measurements:

→ Medipix 1 and 2

Imaging modalities

A. monoenergetic projective image

B. spectral projective image

C. tomographic image (3D)

A. Monoenergetic Projective Image

Intensity at detector position (x,y) :

$$I(x,y) = I_o e^{-\int_0^L \mu(\vec{r})\, dl}$$

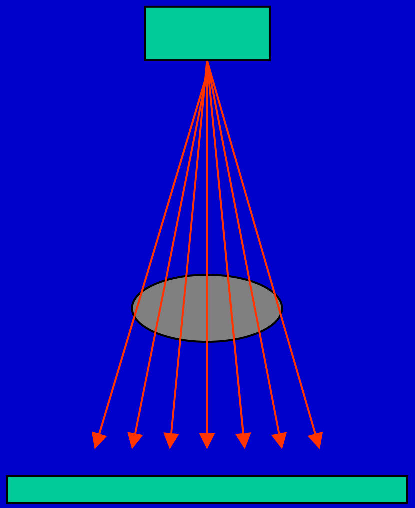


Image information at position (x,y) :

$$t(x,y) = -\ln \frac{I(x,y)}{I_o}$$

$$= \int_0^L \mu(\vec{r})\, dl$$

$$\mu = \mu(E)$$

Aim: Visibility of Structures

Structure: $$I_1(x_1, y_1) \neq I_2(x_2, y_2)$$

Detector requirements:

direct:

position resolution	p (first interaction)
intensity resolution	c = max

indirect:

pixel size:	→ 0
detection efficiency	ε=1
high dynamic range	0, inf
low noise	noise = 0

Position resolution

ideal: measure point of first interaction with infinite resolution

real: 1) physics of X-Ray interaction with sensor material

first interaction:

photoabsorption. Compton scattering, Rayleigh scattering

e^- $e^- \gamma$ $e^- \gamma$ γ

secondary interactions:

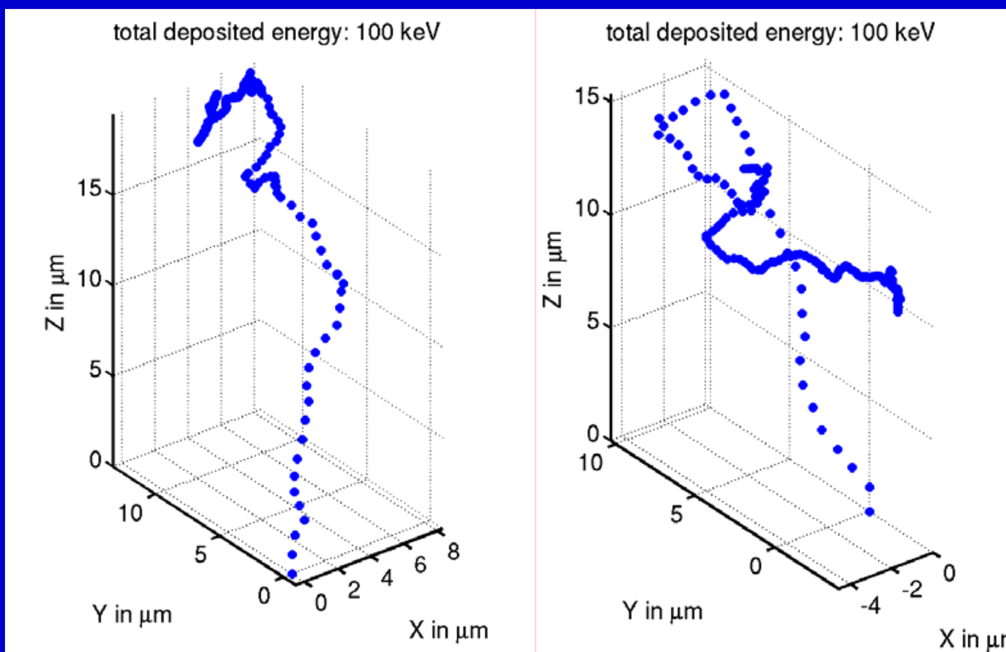
electrons: dE/dx, photons: photoabs., Compton, Rayleigh

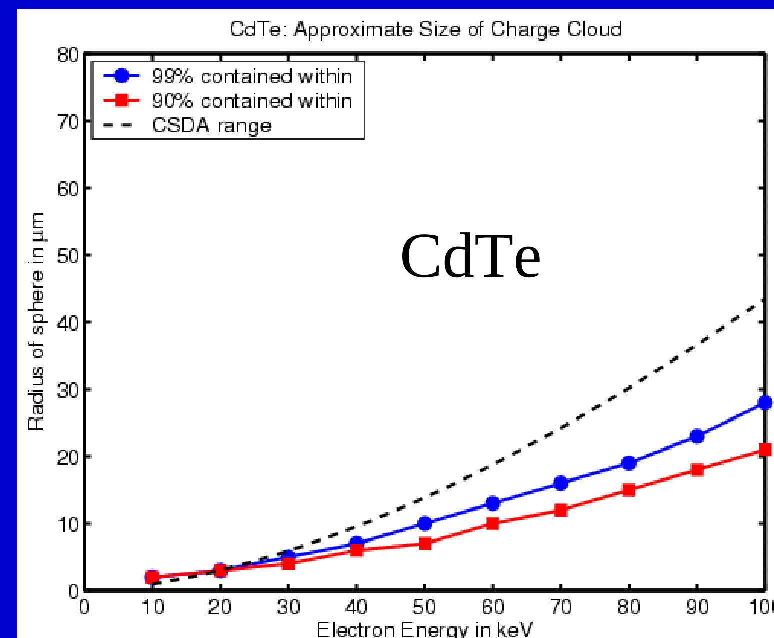
2) propagation of charges (scin.photons)

3) finite size of sensor (read out) pixels

1) physics of X-Ray interaction with sensor material

Propagation of electrons:



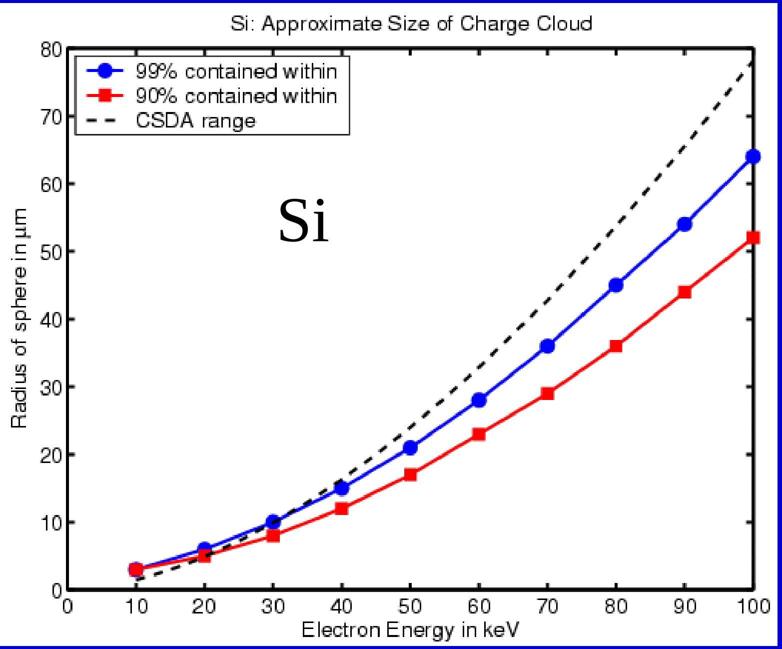


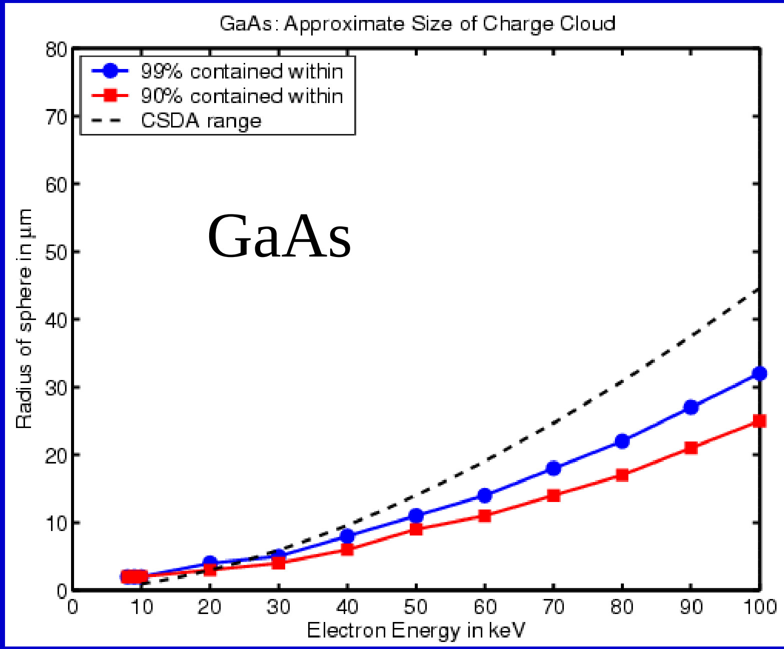
Electron 90%energy range: < 20 µm for E < 100 keV in CdTe

plots taken from the phd thesis of M.Mitschke, Erlangen 2005

1) physics of X-Ray interaction with sensor material

Propagation of electrons:





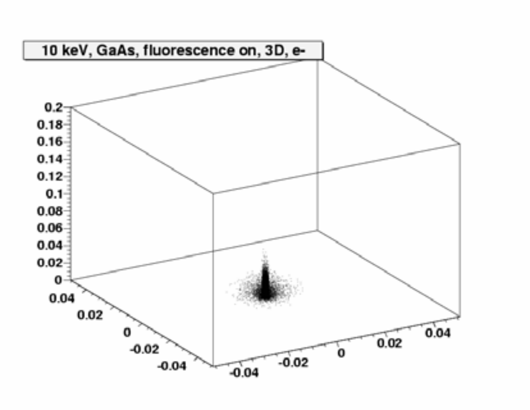
Electron 90% energy range: < 50 µm for E < 100 keV in Si

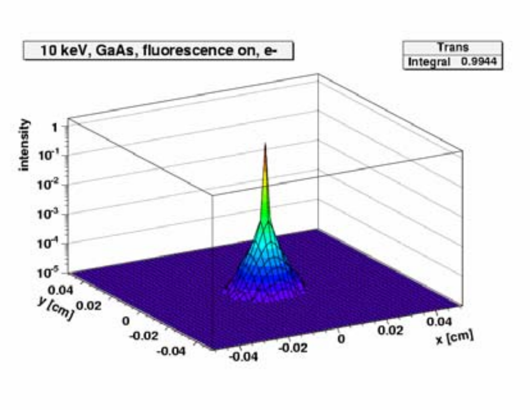
plots taken from the phd thesis of M.Mitschke, Erlangen 2005

1) physics of X-Ray interaction with sensor material

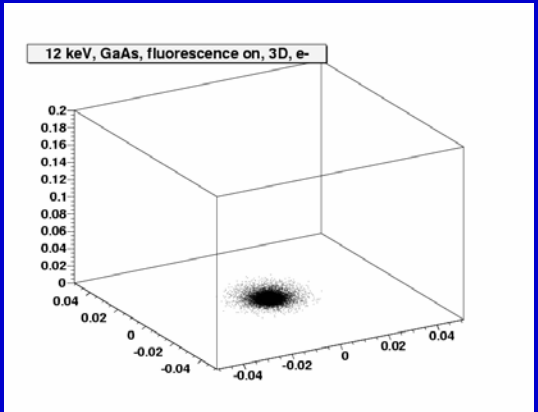
Propagation of secondary photons:

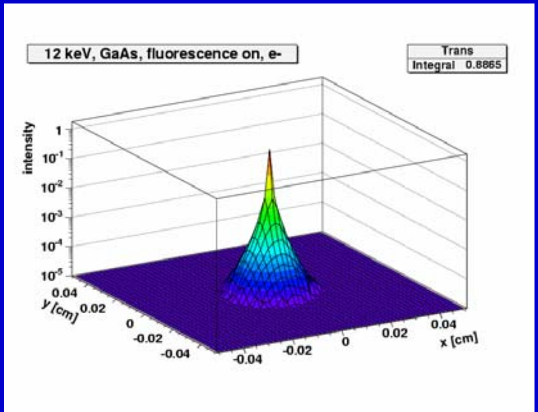
GaAs
10 keV





GaAs
12 keV

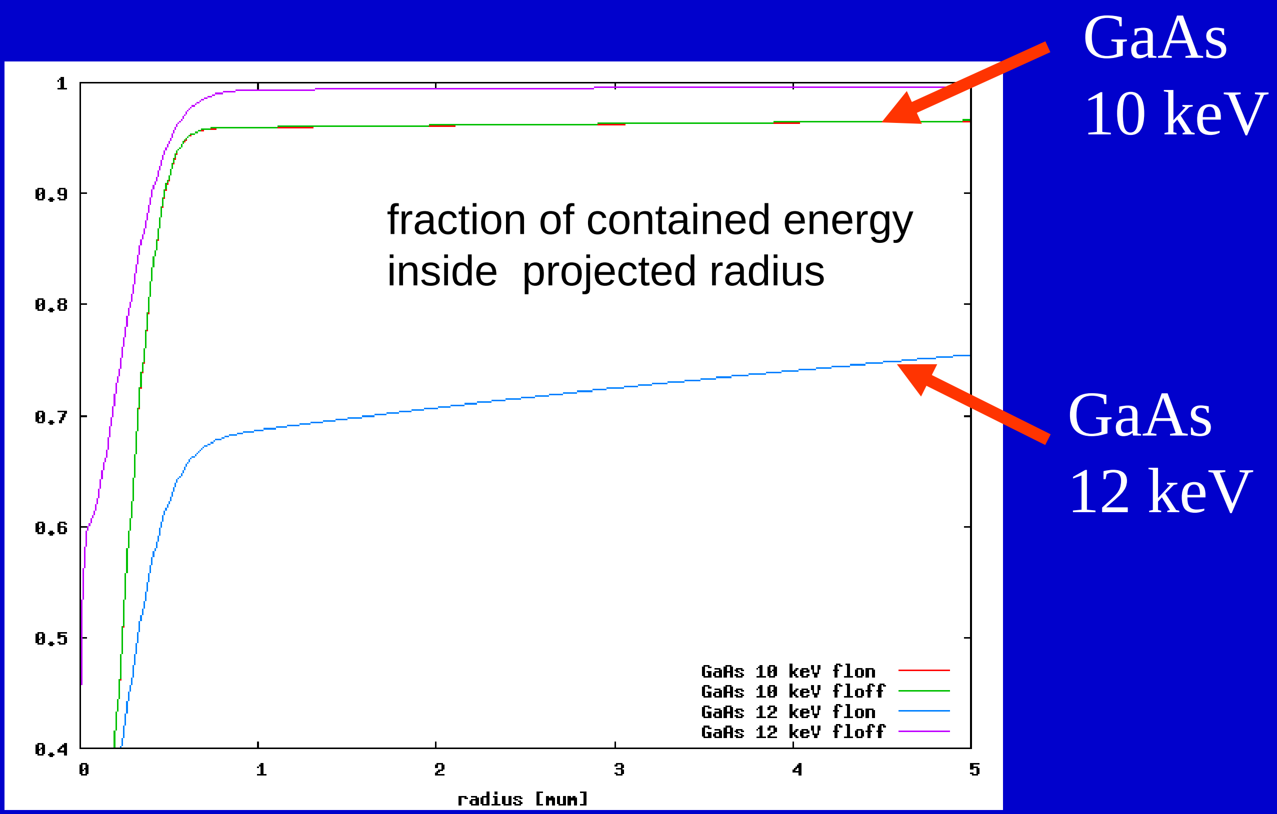




plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

Propagation of secondary photons:

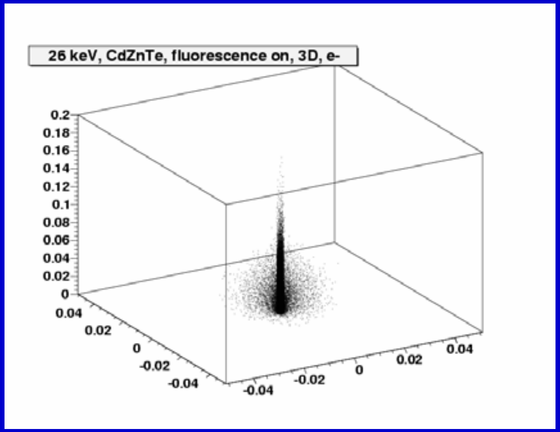


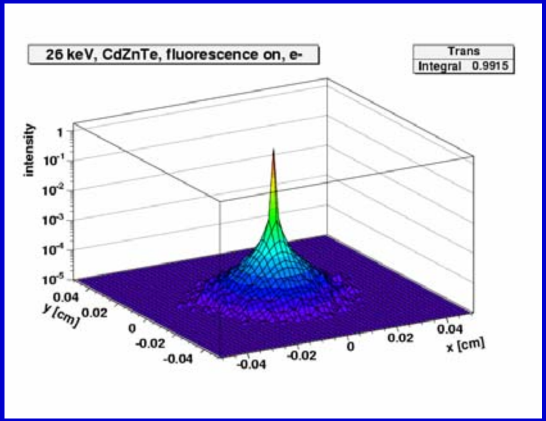
plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

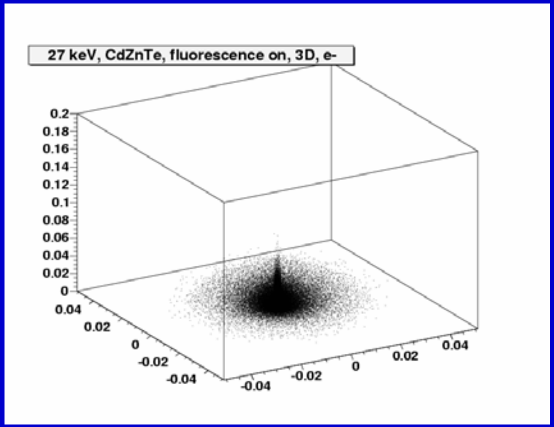
Propagation of secondary photons:

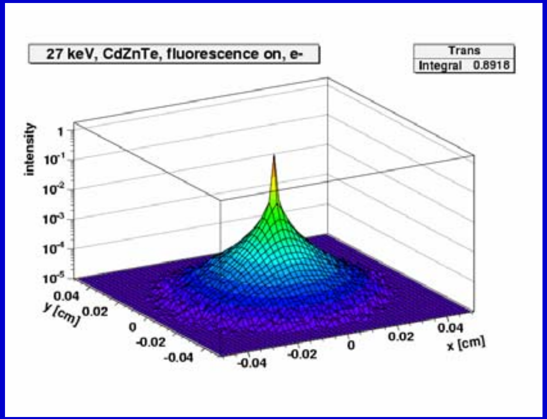
CZT
26 keV





CZT
27 keV

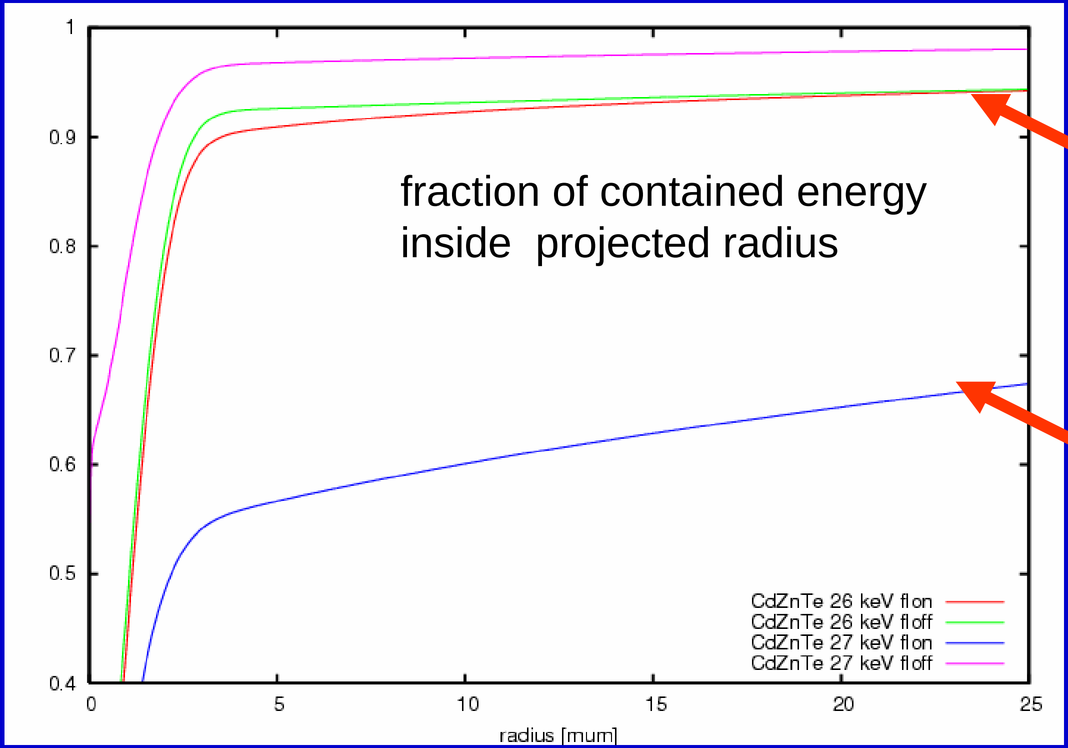




plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

Propagation of secondary photons:



CZT
26 keV

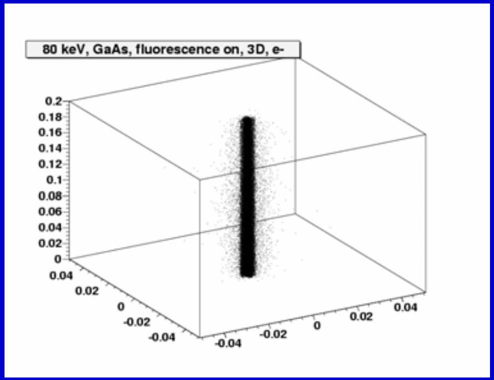
CZT
27 keV

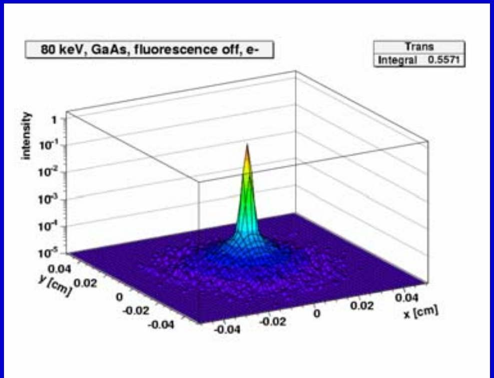
plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

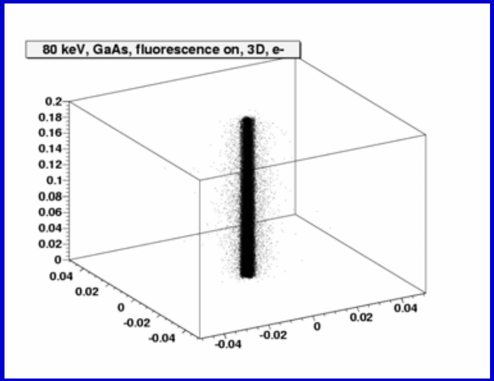
Propagation of secondary photons:

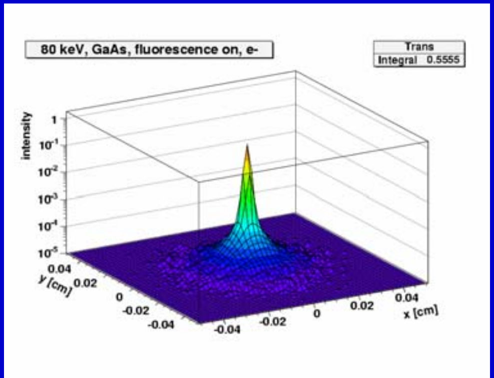
GaAs; 80 keV
fluorescence off





GaAs; 80 keV
fluorescence on



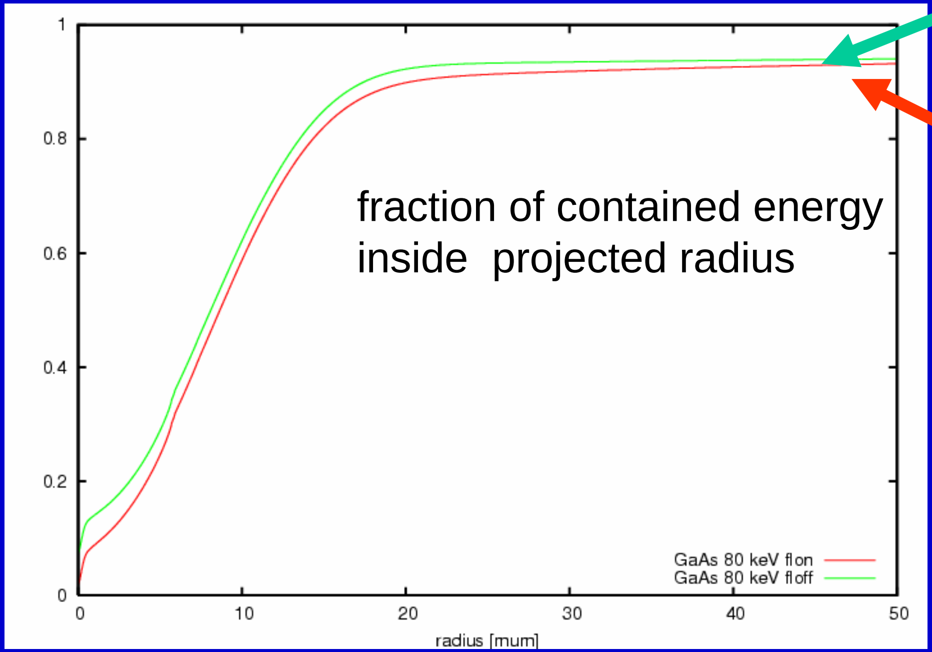


plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

Propagation of secondary photons:

GaAs; 80 keV
fluorenscence off



GaAs; 80 keV
fluorenscence on

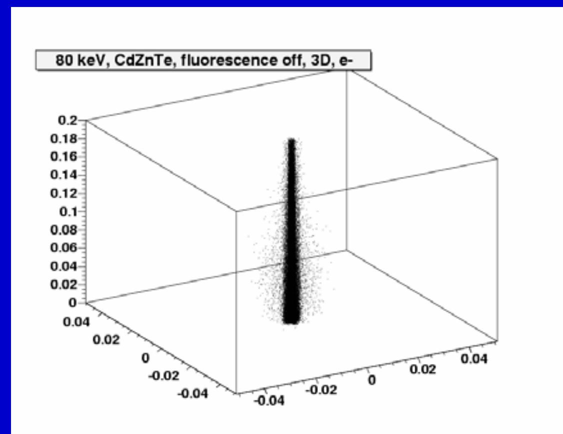
Radius of 90 % of deposited Energy: r < 20 μm for E < 80 keV in GaAs

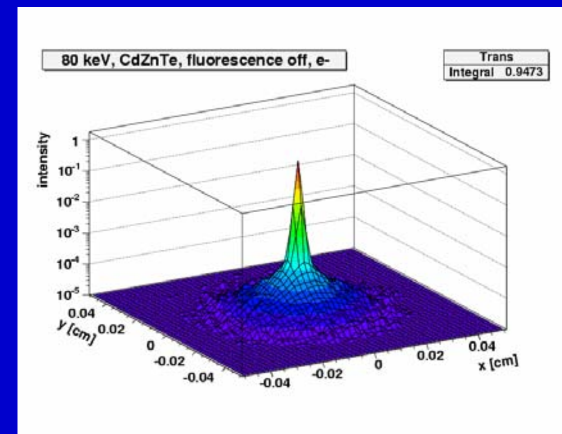
plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

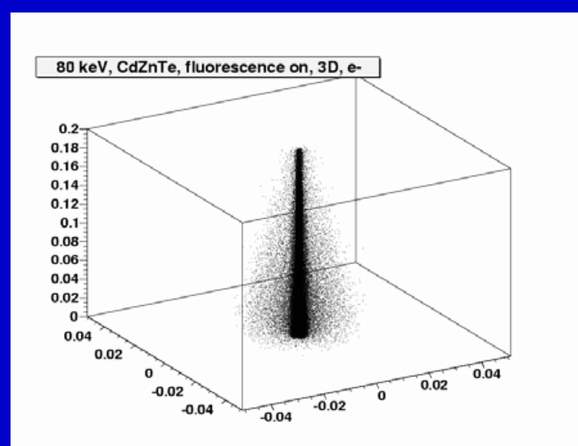
Propagation of secondary photons:

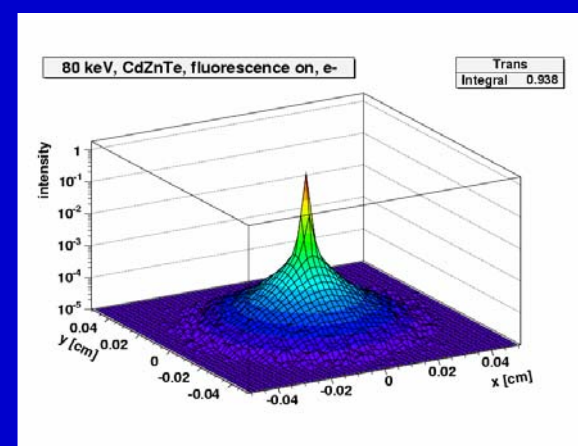
CdZnTe; 80keV
fluorescence off





CdZnTe; 80keV
fluorsecence on



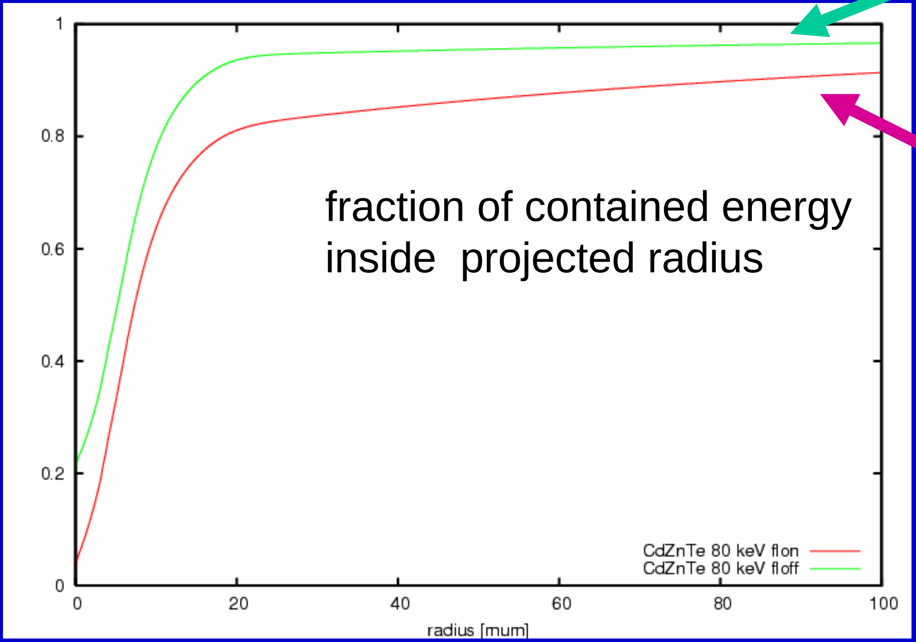


plots taken from the work of J.Durst, Erlangen 2004

1) physics of X-Ray interaction with sensor material

Propagation of secondary photons:

CZT; 80 keV
fluorenscence off



CZT; 80 keV
fluorenscence on

Radius of 90 % of deposited Energy: r < 70 µm for E < 80 keV in CdZnTe

plots taken from the work of J.Durst, Erlangen 2004

2) propagation of charges: losses and charge sharing

Propagation of electrons and holes in semiconductor sensors:

Important parameters:
sensor bias voltage --> electrical field strength
lifetime of electrons and holes
mobility of electrons and holes
homogeneity of material

see talk by Michaela Mitschke

result: 90 % charge radius r < 10 µm for E < 100 keV in 300 µm CdZnTe

Spatial energy deposition and charge sharing lead to:

For counting detectors:

--> enlarged effective pixel size for low threshold
reduced effective pixel size for high threshold
-->see talk by Michaela Mitschke

--> reduced counting efficiency for high threshold

--> double counting for low threshold

for intergrating detectors:

enlarged effective pixel size
no false fluence

Summary: contributions to position resolution

real: 1) physics of X-Ray interaction with sensor material

photoabsorption. Compton scattering, Rayleigh scattering

e^- e^- γ e^- γ γ

90 % energy radius: $r < 10 – 100$ μm

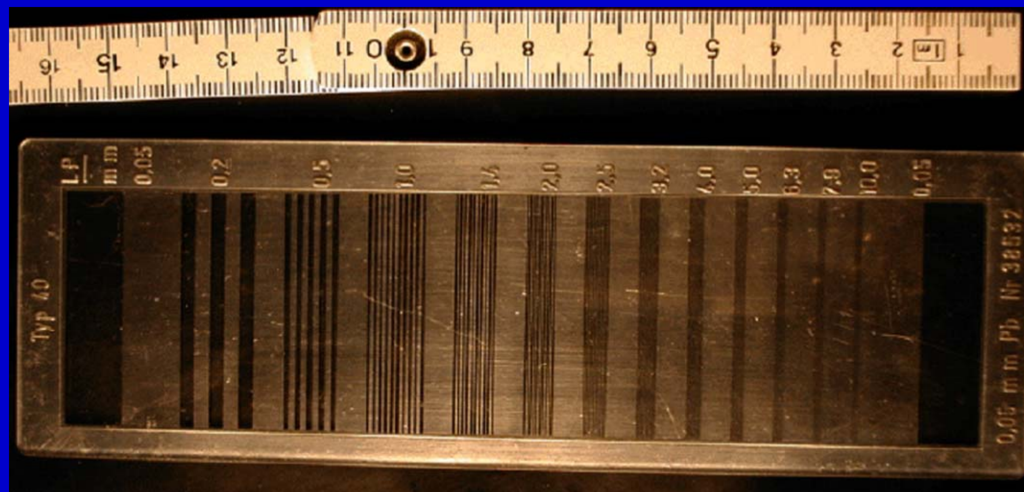
2) propagation of charges:

90 % charge radius: $r < 10$ μm

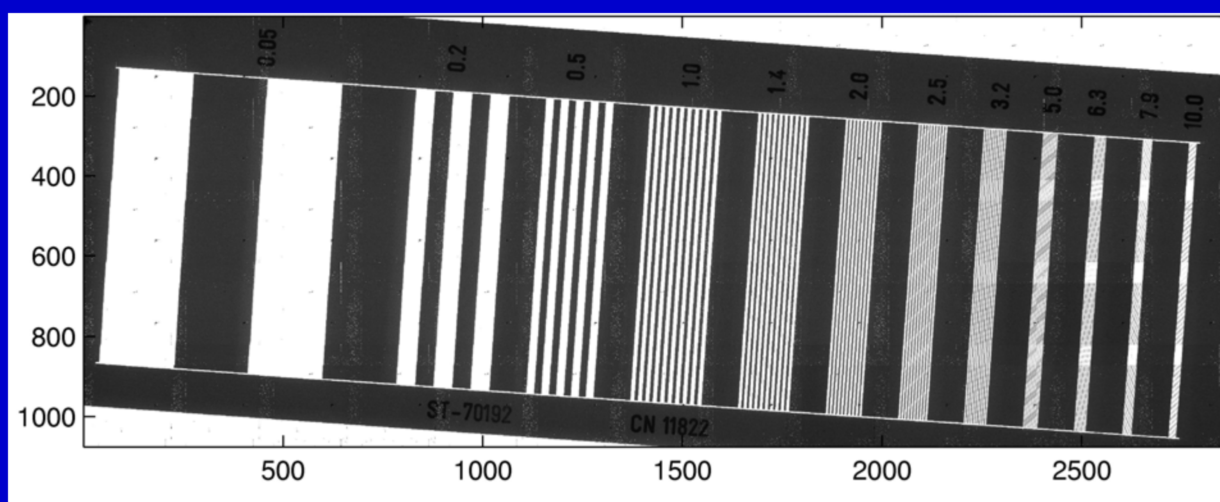
3) finite size of read out pixels: examples:

Medipix 1: 170 μm
Medipix 2: 55 μm

Visibility of structures: Hüttner-Grid



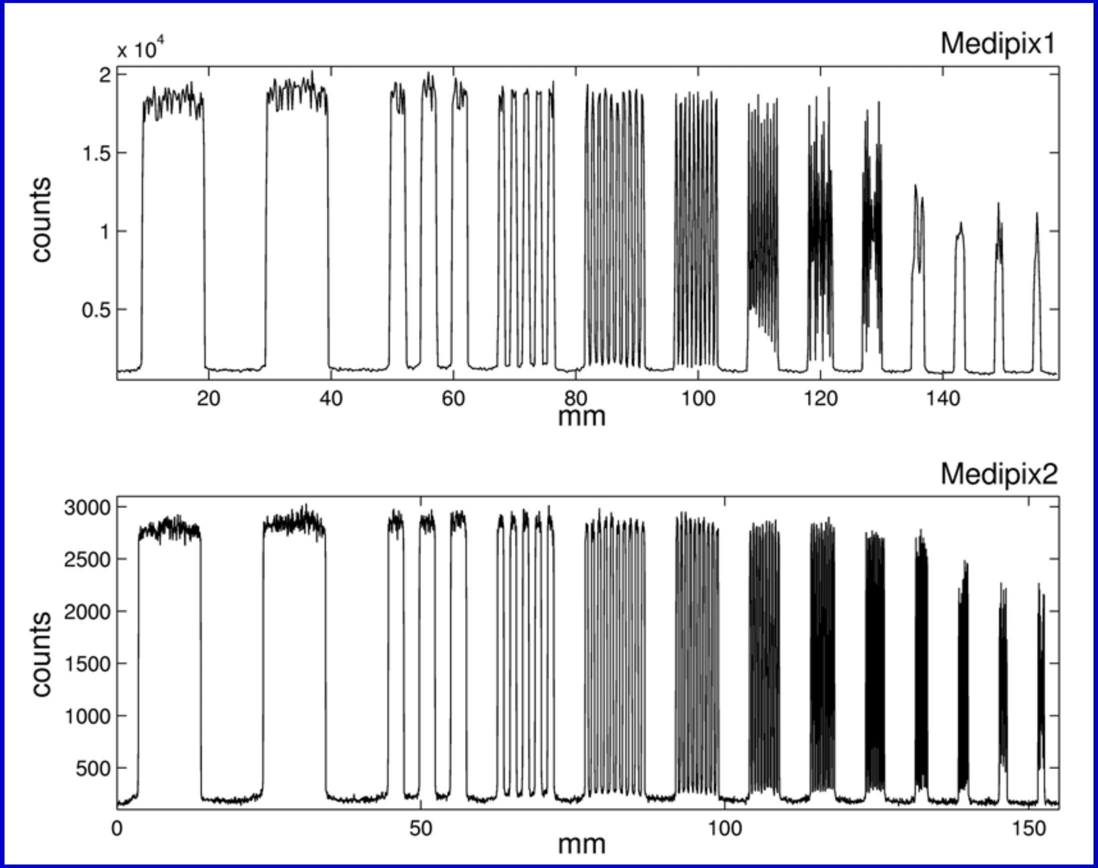
photograph



X-Ray Image (spectral)

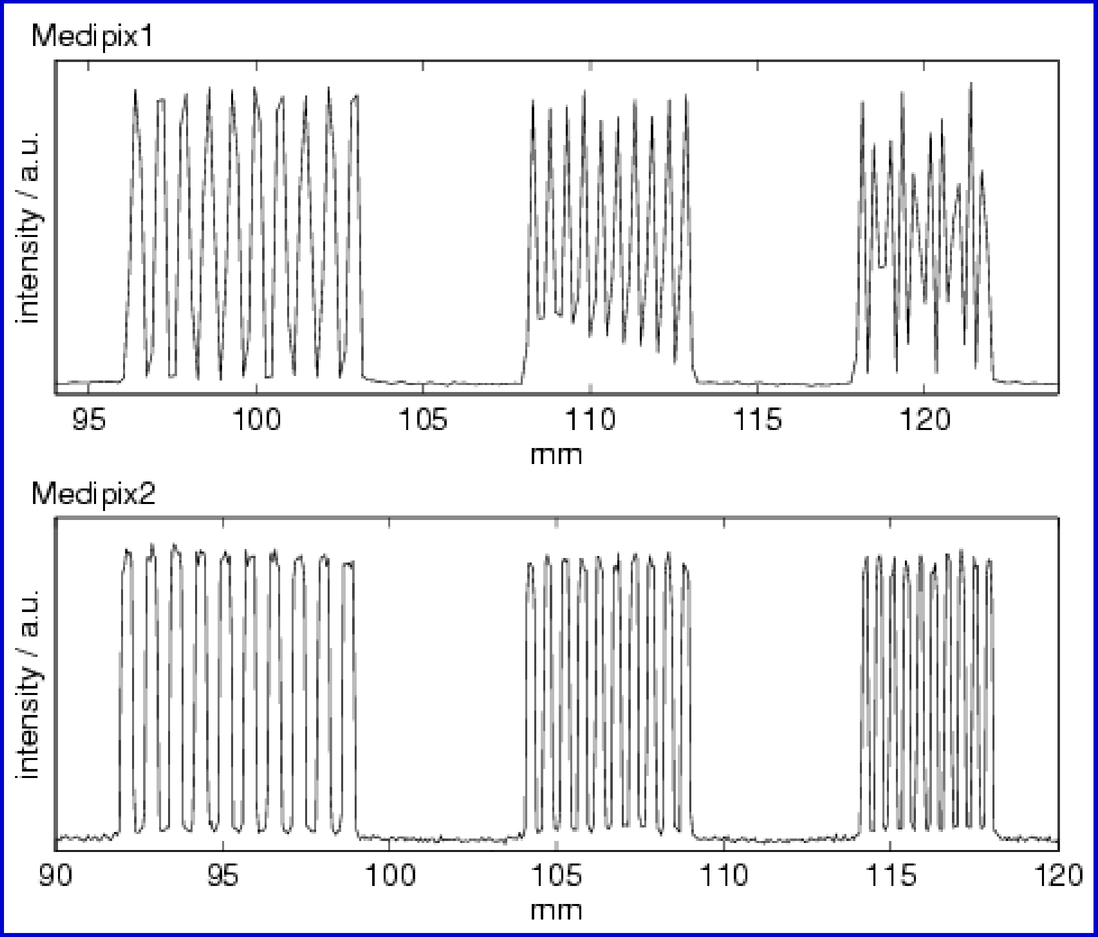
plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Visibility of structures: Hüttner-Grid



plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Visibility of structures: Hüttner-Grid



1.4 lp/mm 2.0 lp/mm 2.5 lp/mm

plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Visibility of structures: Hüttner-Grid

Sampling theorem:

structure size s
needs pixel size $l = s/2$

spacial frequency f
needs sampling frequency: $f_{samp} = 2f$

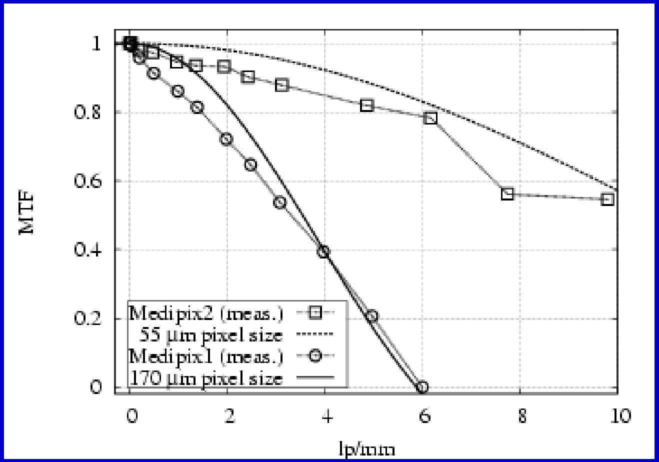
Modulation transfer function:

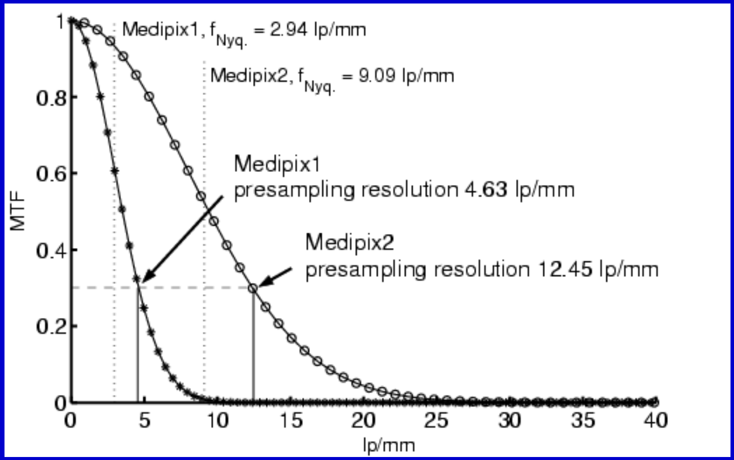
$$MTF(u,v) = \frac{(FT)\ out(x,y)}{(FT)\ in(x,y)}$$

For square pixel:

$$MTF = \frac{\sin(\pi f l)}{(\pi f l)} = \sin c\,(\pi f l)$$

position resolution: MTF with Medipix 1 and 2:





$$MTF(u,v) = \frac{(FT)\ out(x,y)}{(FT)\ in(x,y)}$$

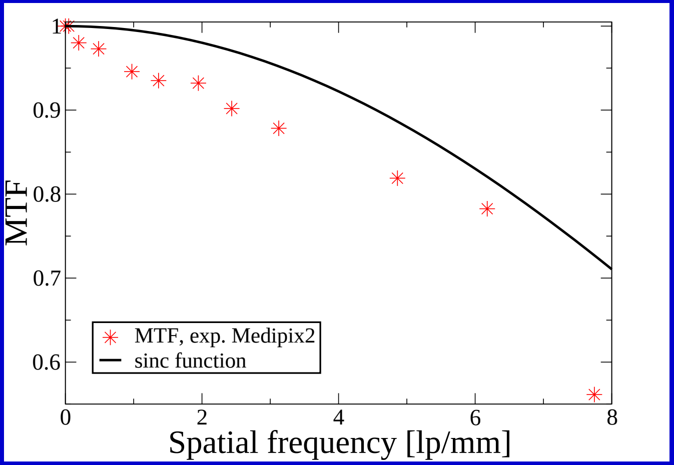
$$MTF\ (u,v) = (FT)\ PSF(x,y)$$

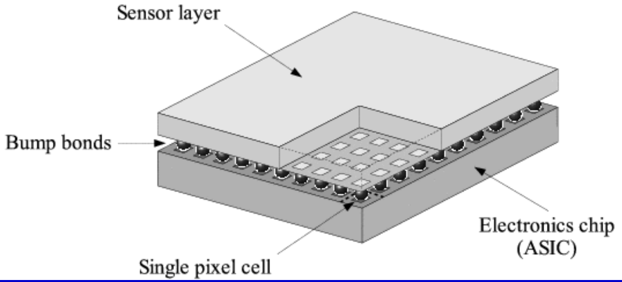
MTF from Hüttner-Grid

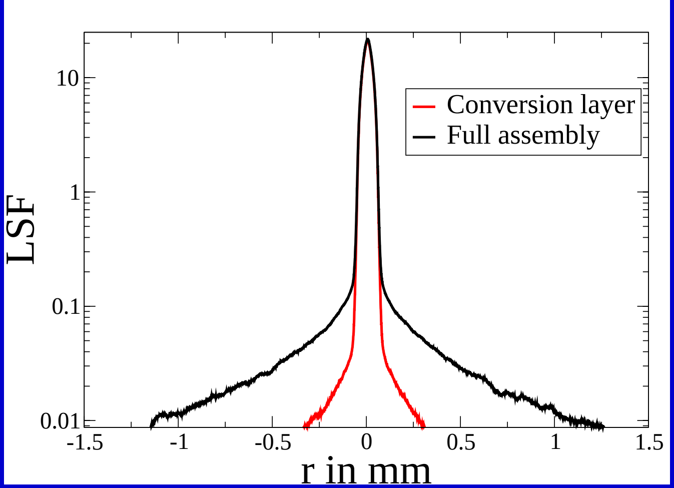
MTF from edge method

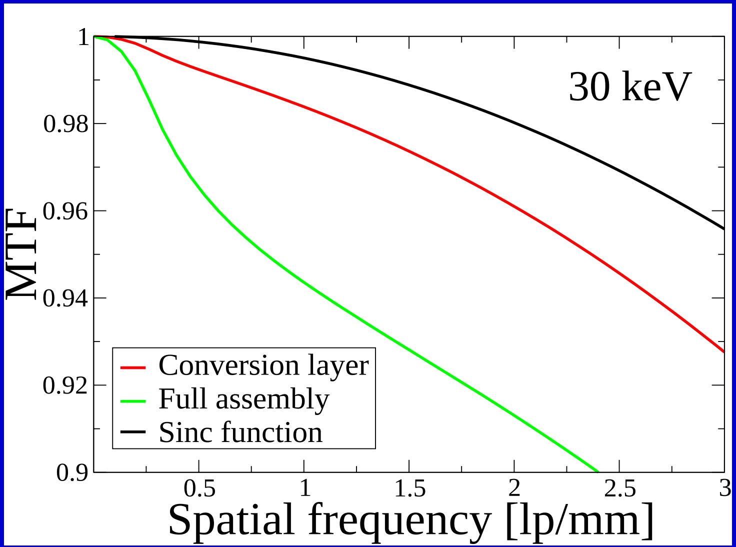
plots taken from the work of F.Pfeiffer, M.Hoheisel, Erlangen 2004

position resolution: MTF and low frequency drop



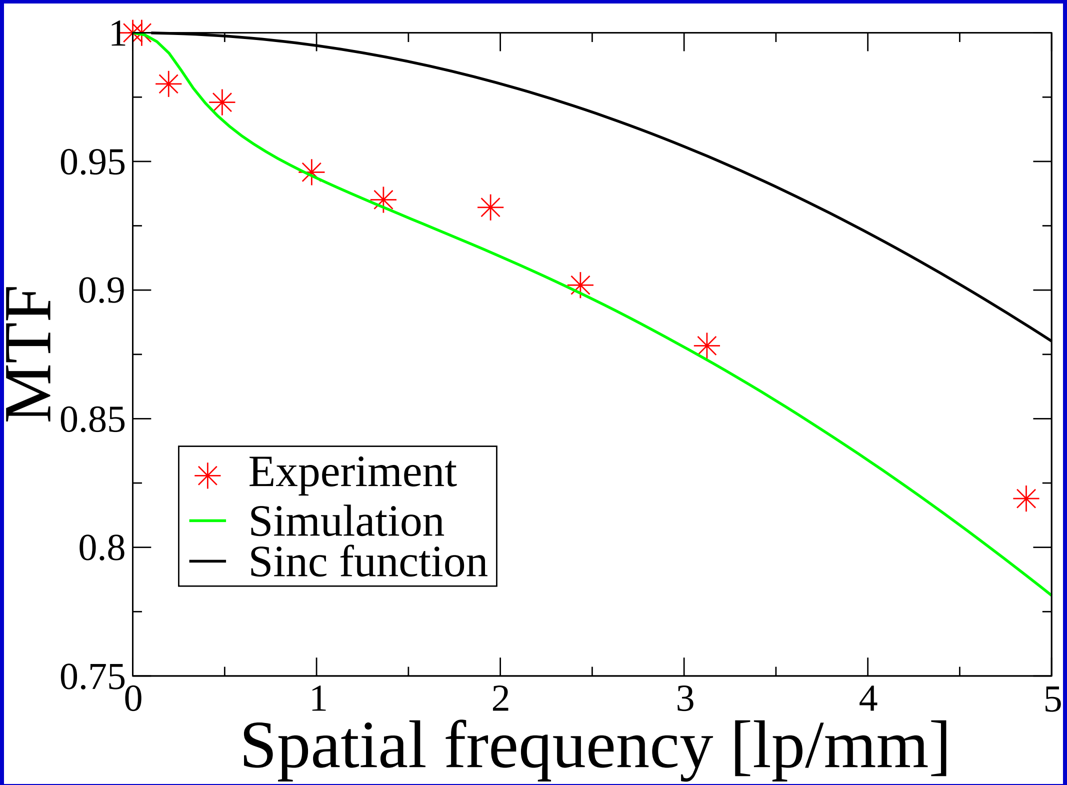






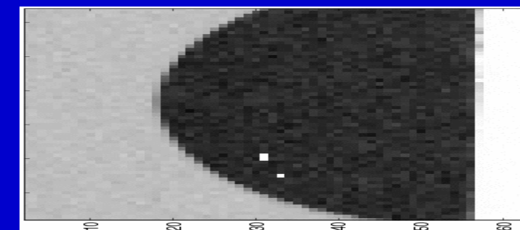
plots taken from the work of A.Korn, M.Hoheisel et al., Erlangen

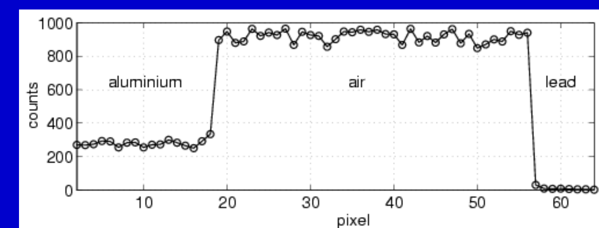
position resolution: MTF and low frequency drop



plots taken from the work of A.Korn, M.Hoheisel et al., Erlangen

Visibility of structures : Contrast





Contrast:

$$C = \frac{I_2 - I_1}{I_1 + I_2} = \frac{N_2 - N_1}{N_1 + N_2}$$

Signal difference to noise ratio:

$$SDNR = \frac{\Delta I}{\sigma_I} = C\sqrt{N_1 + N_2}$$

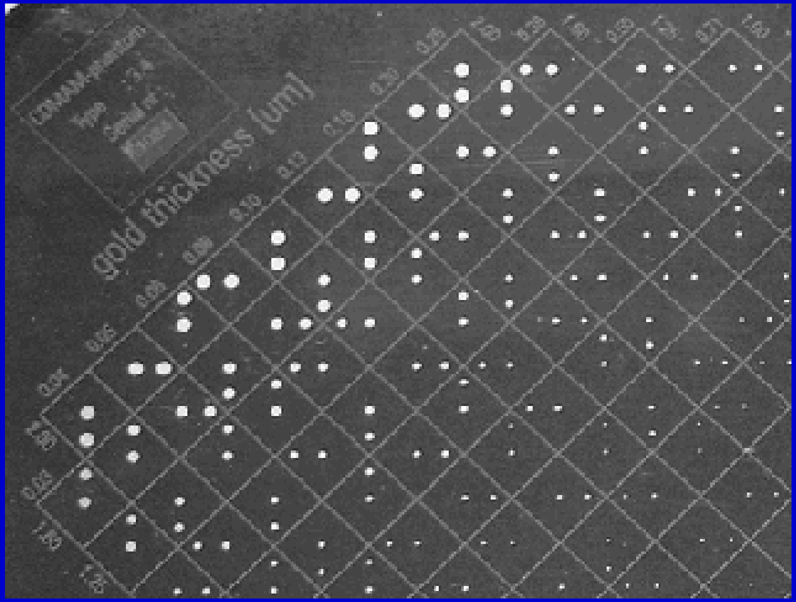
Visibility for given contrast: need large N = Φ A

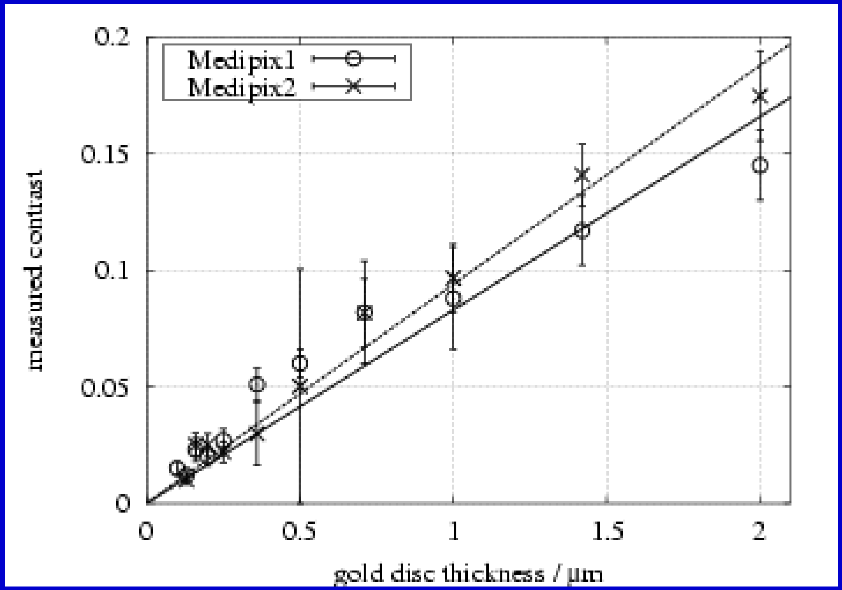
--> long exposure

--> large area of structure

plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Contrast-detail-visibility

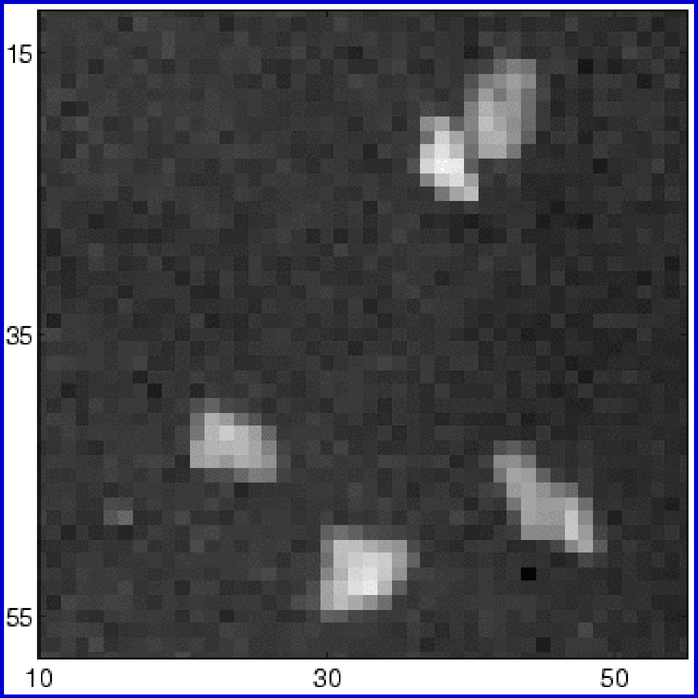




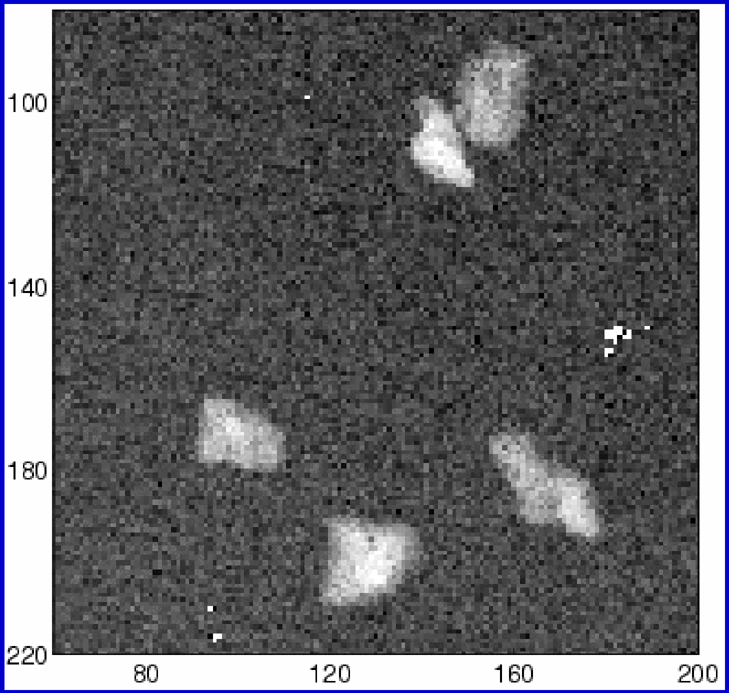
CDMAM phantom: gold disks of various size and thickness

plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Contrast-detail-visibility



Medipix 1

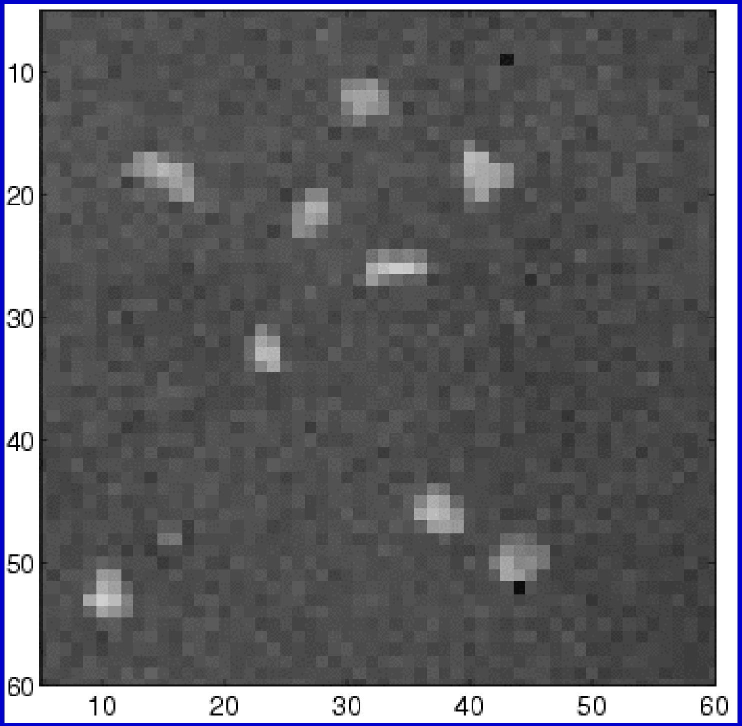


Medipix 2

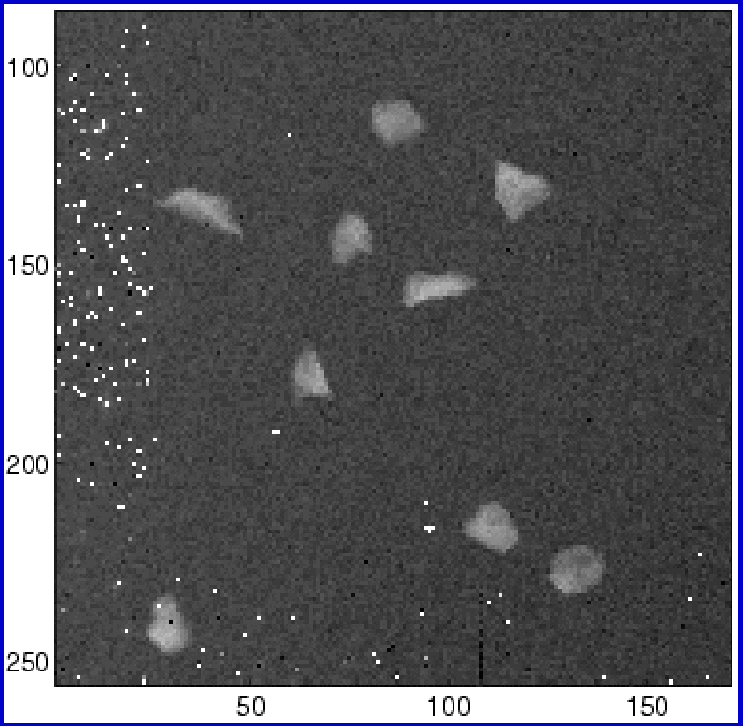
Mammographic phantom: Al_2O_3 grains, size 1.1 to 1.5 mm

plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

Contrast-detail-visibility



Medipix 1

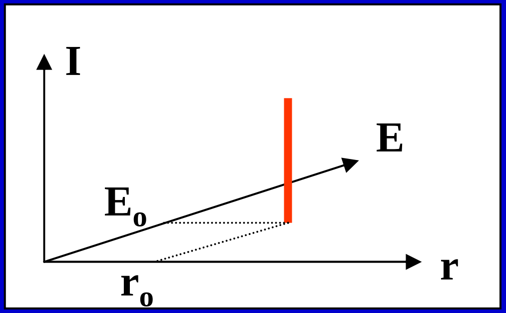


Medipix 2

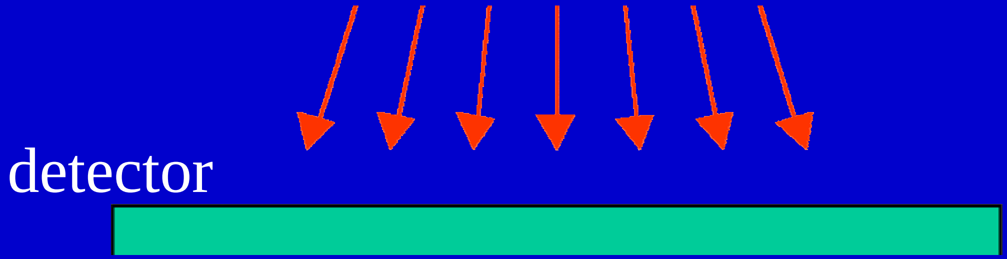
Mammographic phantom: Al_2O_3 grains, size .55 to .75 mm

plots taken from the phd thesis of F.Pfeiffer, Erlangen 2004

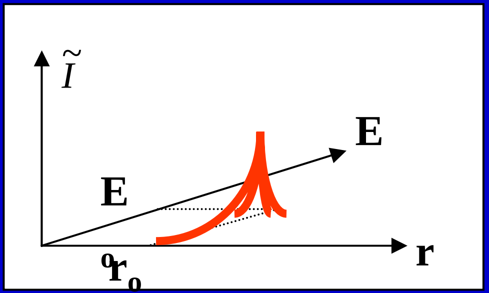
Summary: Detector Transfer Function



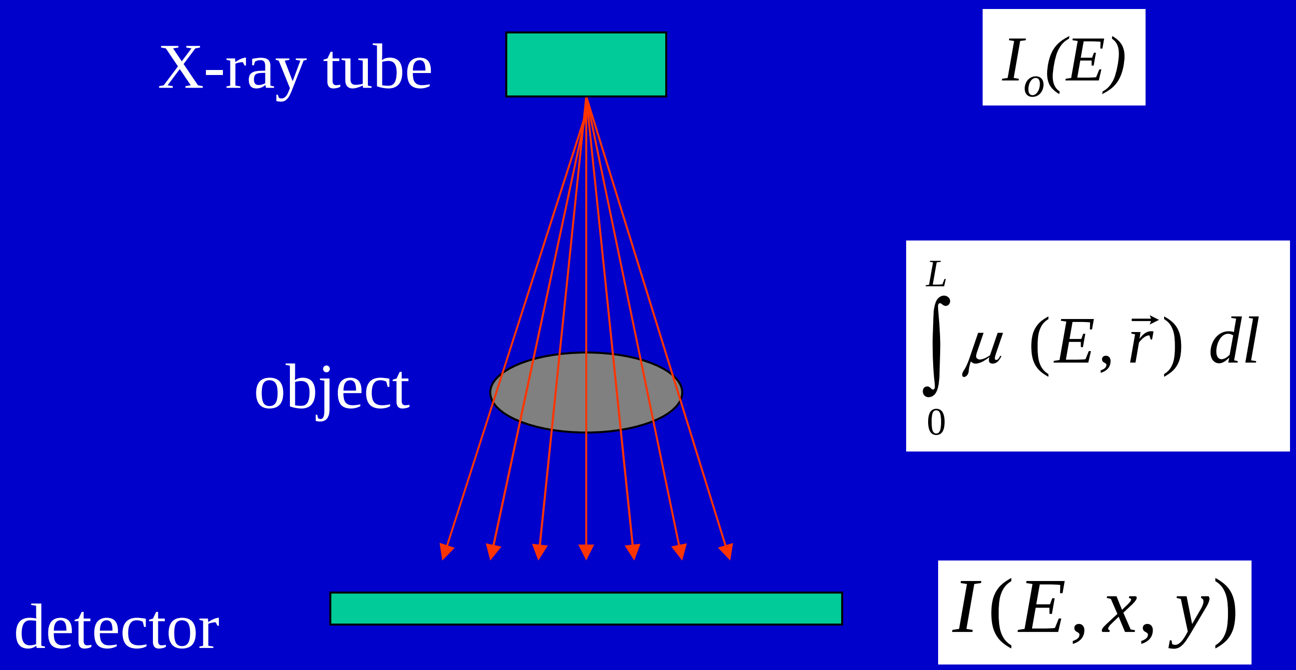
$$I(E_o, x_o, y_o)$$



$$\tilde{I}(E, x, y)$$



B. Spectral Projective Image



Spectral Projective Imaging

Image information:
(energy resolving detector)

$$t(E,x,y) = -\ln \frac{I_o(E)\, e^{-\int_0^L \mu(E,\vec{r})\, dl}}{I_o(E)} = \int_0^L \mu(E,\vec{r})\, dl$$

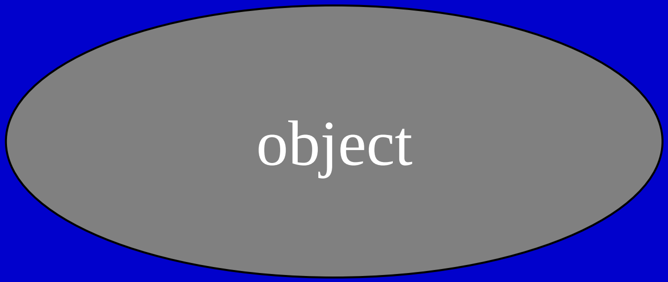
Integrating detector:

$$t_I(x,y) = -\ln \frac{\int_{E_{th}}^{E_{\max}} I_o(E)\, E\, e^{-\int_0^L \mu(E,\vec{r})\, dl}\, dE}{\int I_o(E)\, E\, dE}$$

Counting detector:

$$t_C(x,y) = -\ln \frac{\int_{E_{th}}^{E_{\max}} I_o(E)\, e^{-\int_0^L \mu(E,\vec{r})\, dl}\, dE}{\int I_o(E)\, dE}$$

Spectral projective X-Ray Image Information

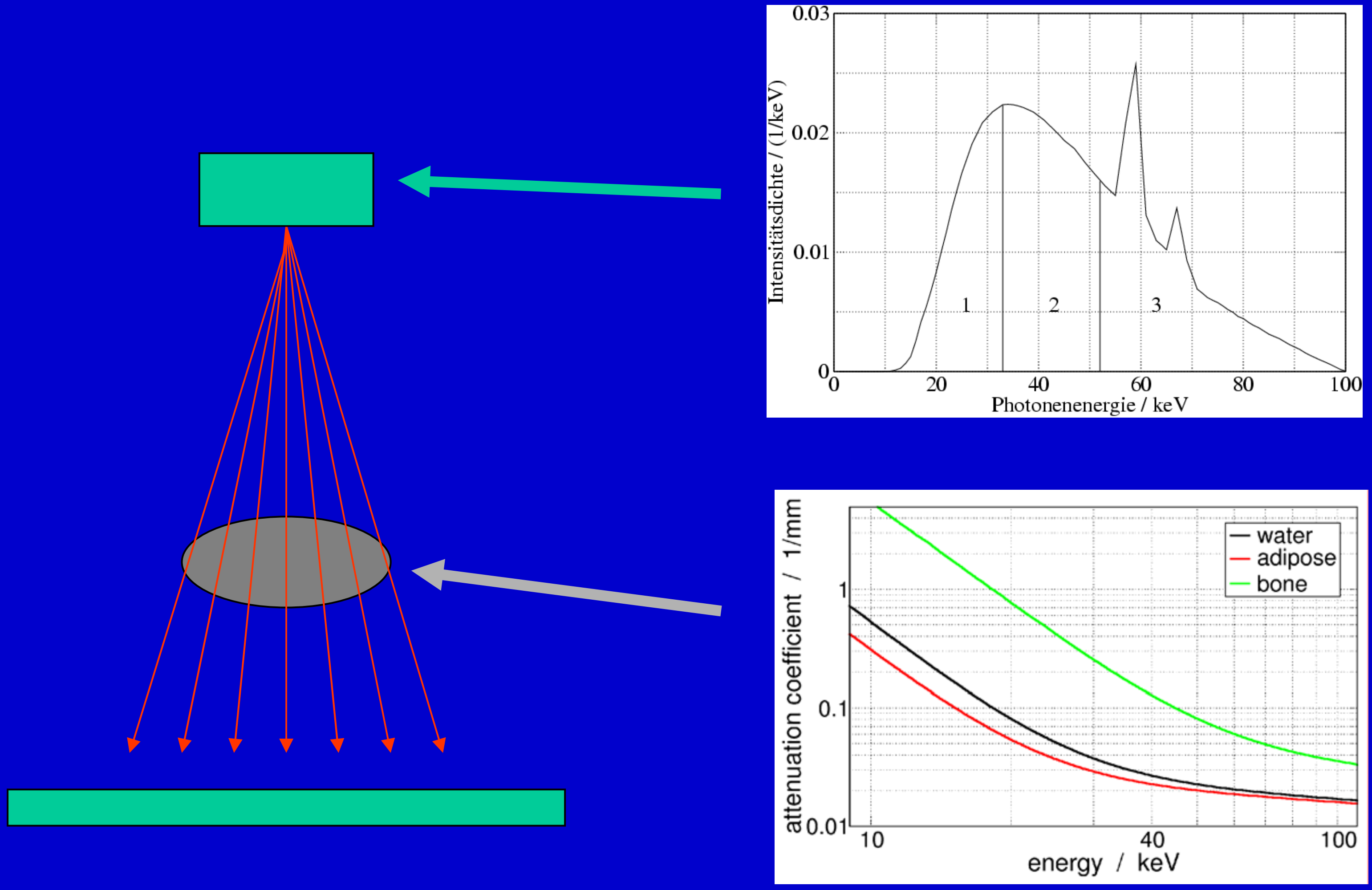


$$\int_0^L \mu\,(E,\vec{r})\,dl$$

How to make use of the energy information:

1) single image for each energy --> too much noise per image !
2) contrast energy weighted image
3) material reconstruction

Spectral sensitivity



2. Contrast energy weighted image

Contrast:

$$C(E_i) = \frac{I_2(E_i) - I_1(E_i)}{I_2(E_i) + I_1(E_i)} = \frac{e^{-\mu_{1i}d_1} - e^{-\mu_{2i}d_2}}{e^{-\mu_{1i}d_1} + e^{-\mu_{2i}d_2}} = w_i$$

By taking

$$I = \sum I_i w_i$$

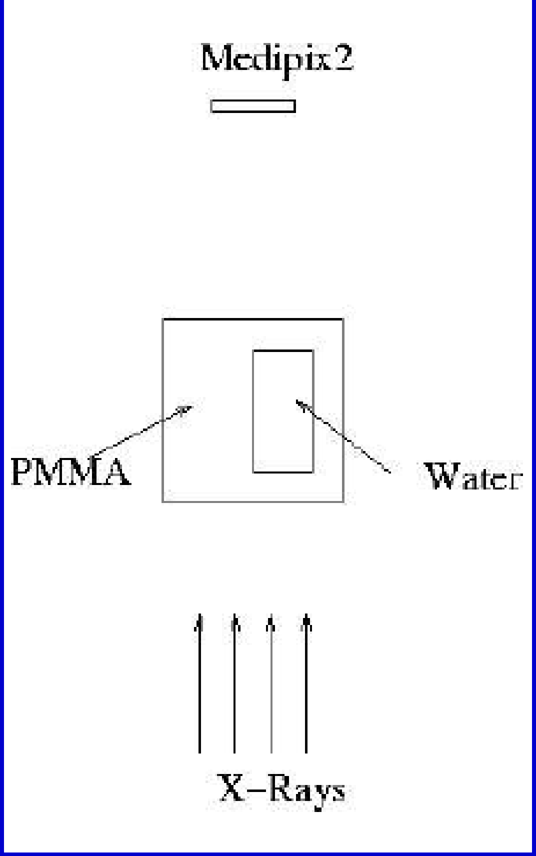
SDNR=max :

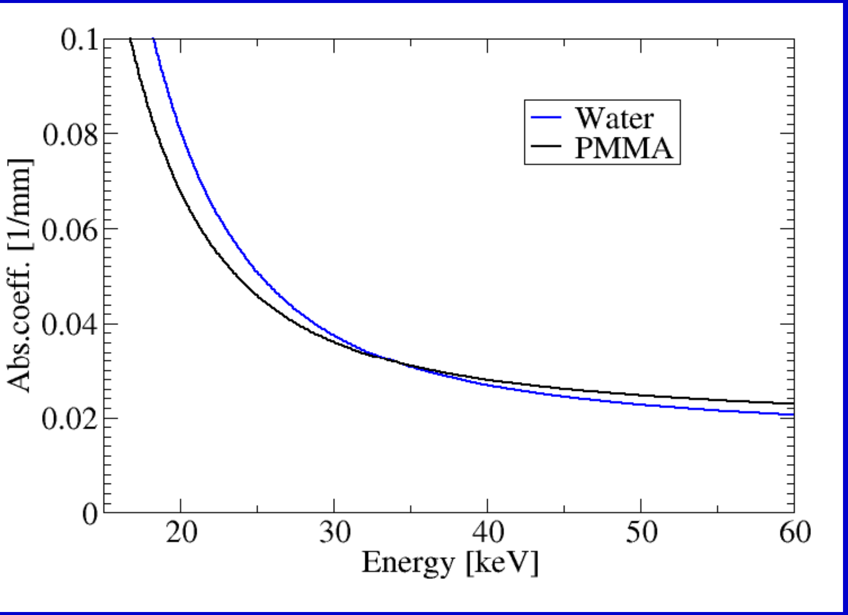
$$SDNR = \frac{I_2 - I_1}{\sigma} = \frac{\sum (I_{2,i} - I_{1,i}) w_i}{\sqrt{\sum (I_{2,i} + I_{1,i}) w_i^2}}$$

Quality:

$$FOM = \frac{SDNR^2}{dose}$$

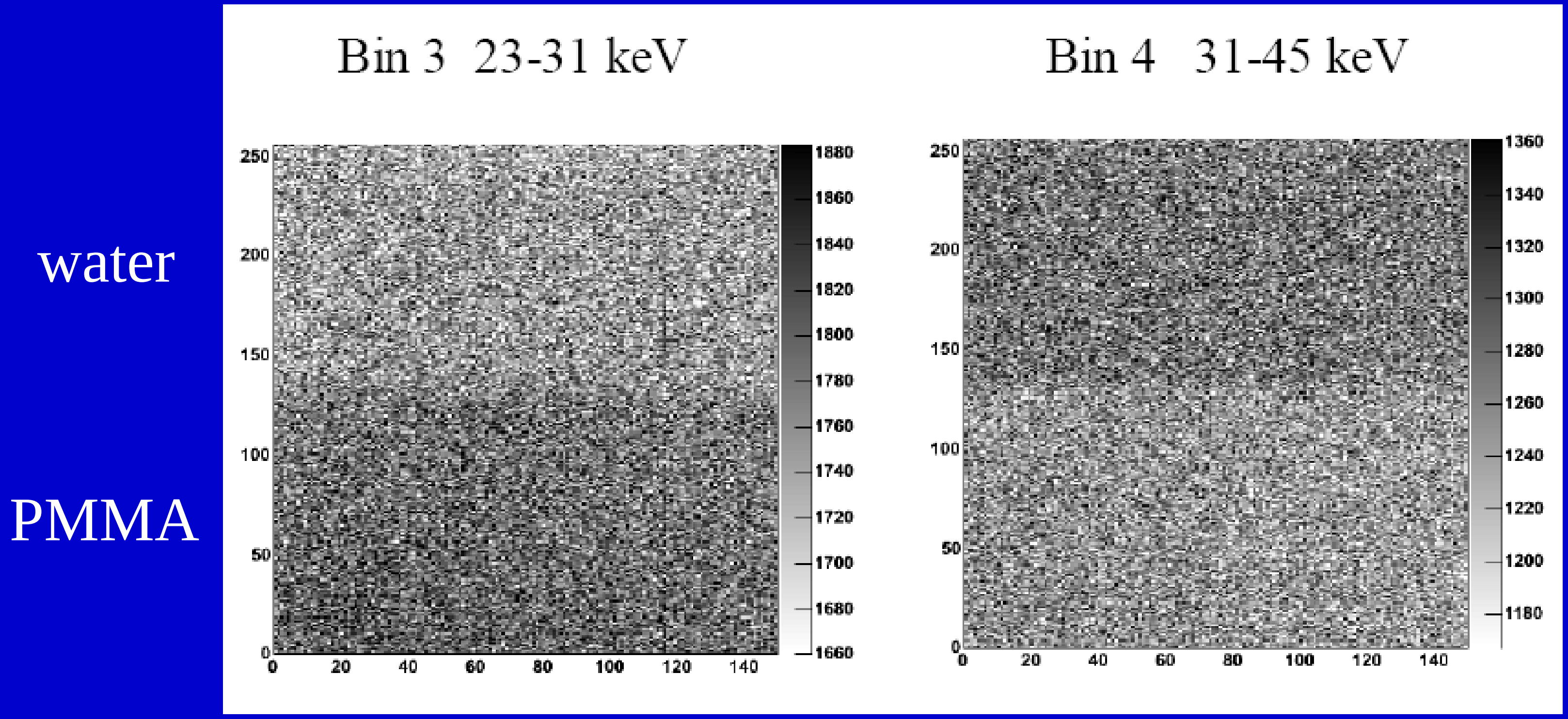
Examples of energy weighted images





60 kV, W anode ,2mm Al filter, 300 mu Si, 150 V

Images in energy intervals



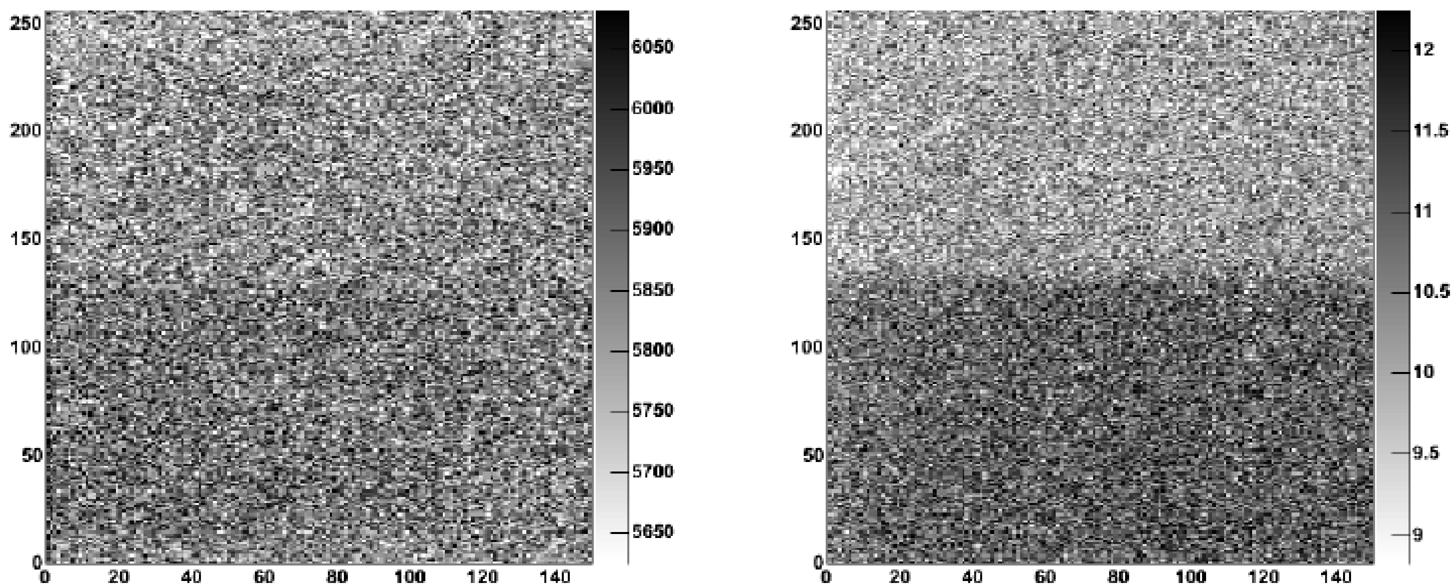
60 kV, W anode ,2mm Al filter, 300 mu Si, 150 V

plots taken from the diploma thesis of J.Karg, Erlangen 2004

Examples of energy weighted images

water

PMMA



SNR improvement: > 2.2

photon counting image

energy weighted image

60 kV, W anode ,2mm Al filter, 300 mu Si, 150 V

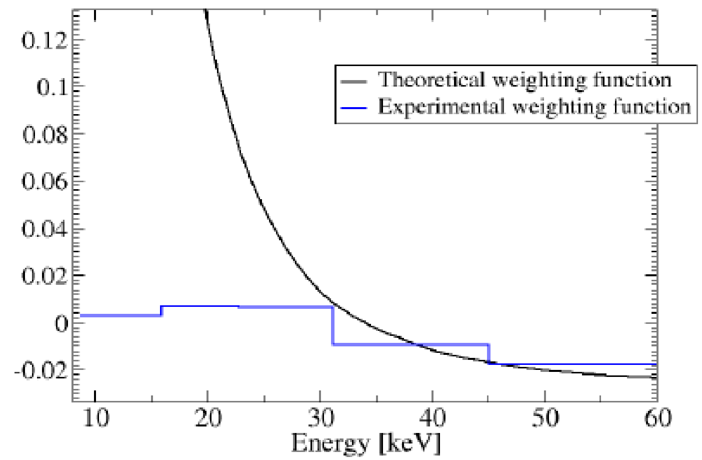
plots taken from the diploma thesis of J.Karg, Erlangen 2004

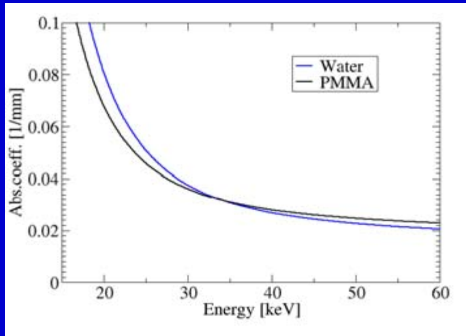
Examples of energy weighted images

- Weighting Function calculated from image:

$$w = \frac{(I_1 - I_2)}{(I_1 + I_2)}$$

- Large discrepancies compared to theoretical weighting function (ideal detector)





Problem with Medipix 2: bad energy resolution due to broad energy deposition and charge sharing !!

--> Medipix 3

3 .Material reconstruction

$$t(E) = -\ln T(E) = -\ln \frac{N(E)}{N_o(E)} = \sum_{j=1}^{m} (\frac{\mu(E)}{\rho})_j \, p_j$$

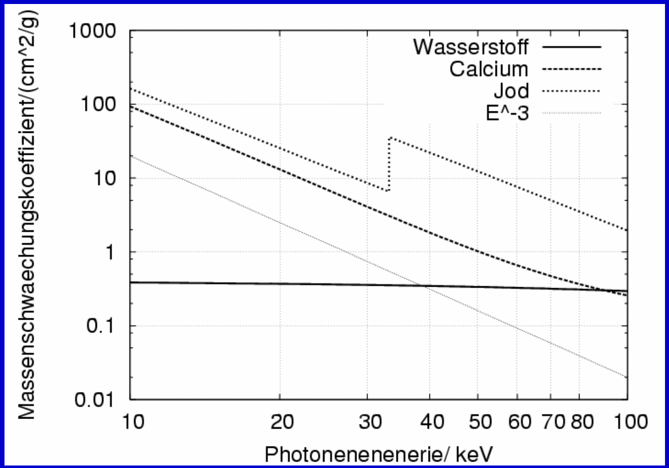
$$p_j = (\rho d)_j$$

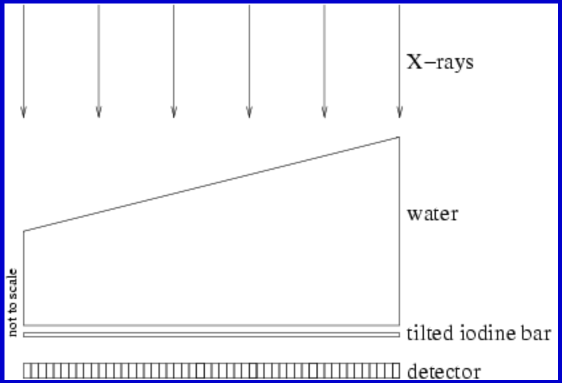
For energy bin i :

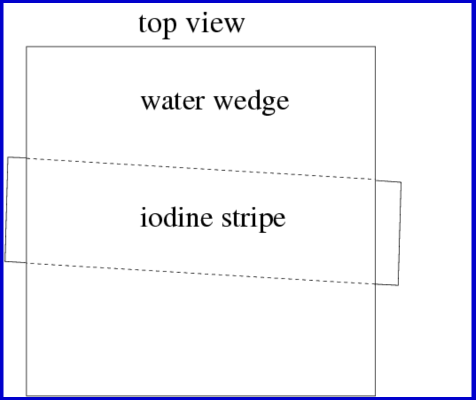
$$t_i = -\ln T_i = -\ln \frac{N_i}{N_{io}} = \sum_{j=1}^{m} (\frac{\mu}{\rho})_{ij} \, p_j = M_{ij} \, p_j$$

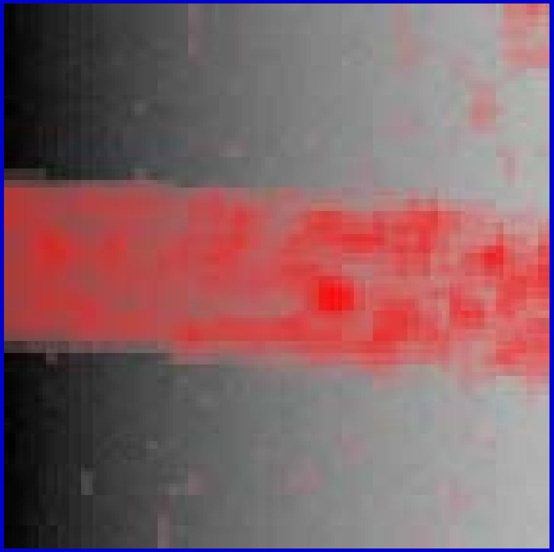
Method: determine for given material matrix M
the amount of material p
which fits best to measured image t

Simulated examples for material reconstruction





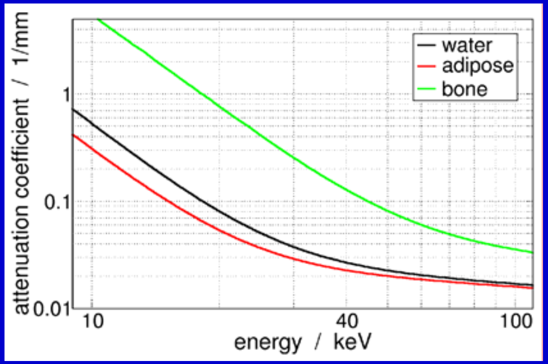




plots taken from the diploma thesis of M.Firsching, Erlangen 2005

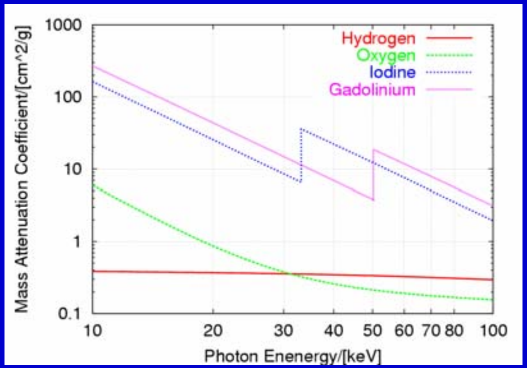
Material reconstruction in medical immaging

Medical imaging:
Most of the contrast is due to density variations:
Atten. coeff. of different „materials“
all have the same energy dependence

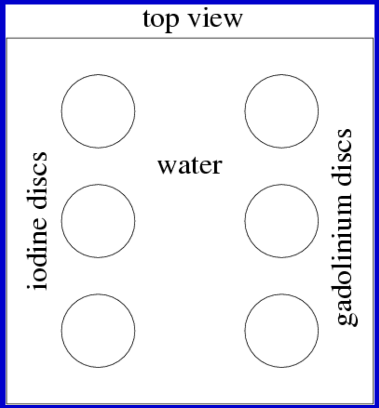


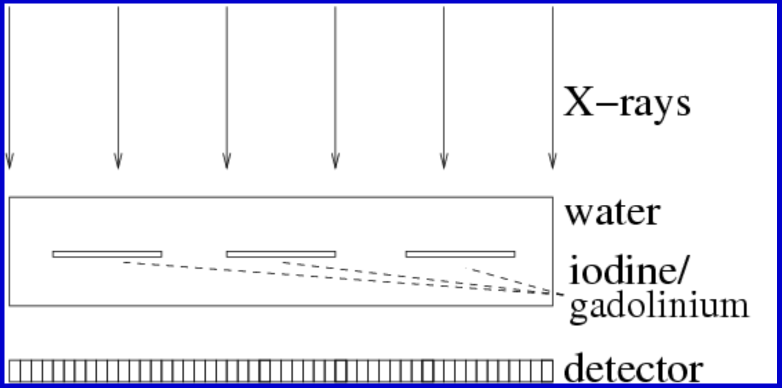
Normal contrast agents have high density.

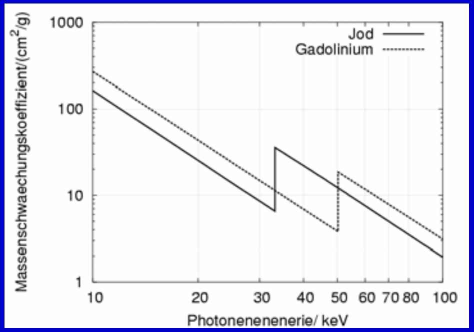
New concept:
Contrast agent has different energy behaviour
→ Quantitative agent reconstruction

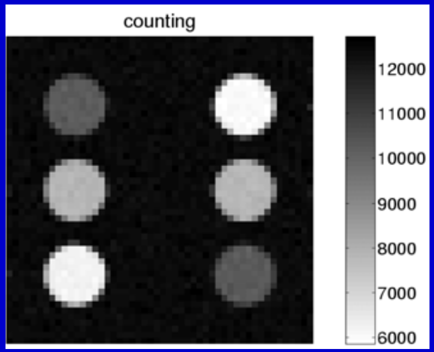


Simulated examples for material reconstruction

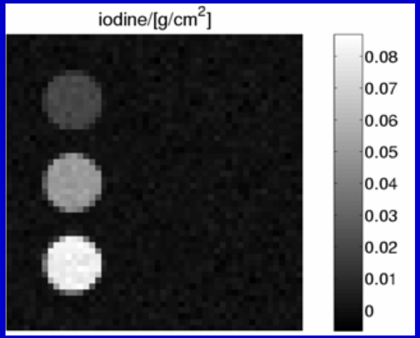




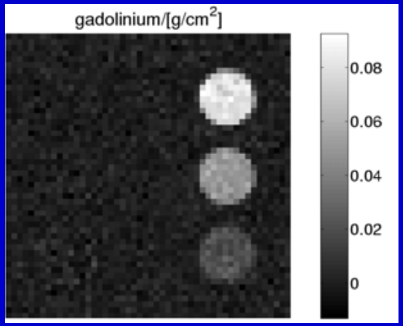




Photon counting



Iodine image



Gadolinium image

plots taken from the diploma thesis of M.Firsching, Erlangen 2005

Material reconstruction for broad energy bins:

$$T_{eff} = \frac{\int_{E_1}^{E_2} T(E)\, S(E)\, dE}{\int_{E_1}^{E_2} S(E)\, dE} = \frac{\int_{E_1}^{E_2} \exp(-\frac{\mu(E)}{\rho}\, p)\, S(E)\, dE}{\int_{E_1}^{E_2} S(E)\, dE}$$

$$\mu_{eff} = -\frac{1}{p} \ln T_{eff}$$

C. Tomographic Images

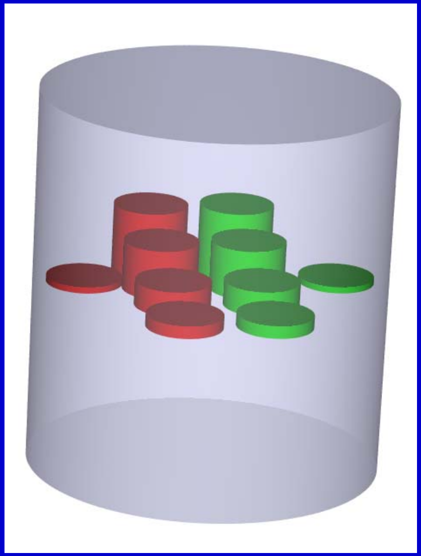
CT-imaging:

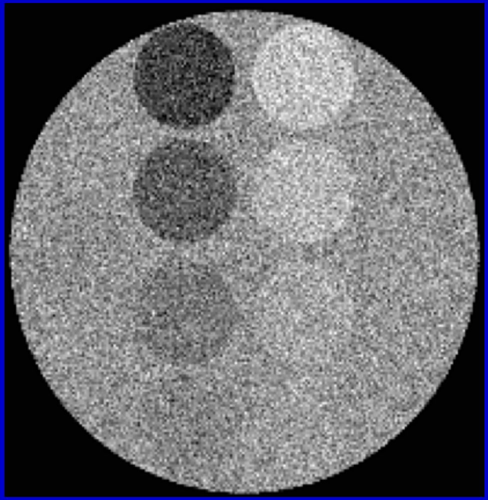
$$t(x,\theta) = -\ln T(x,\theta) = -\ln \frac{N}{N_o}(x,\theta)$$

$$\rightarrow \quad \tilde{\mu}(\vec{r}) = \langle \mu(E,\vec{r}) \rangle_{Det}$$

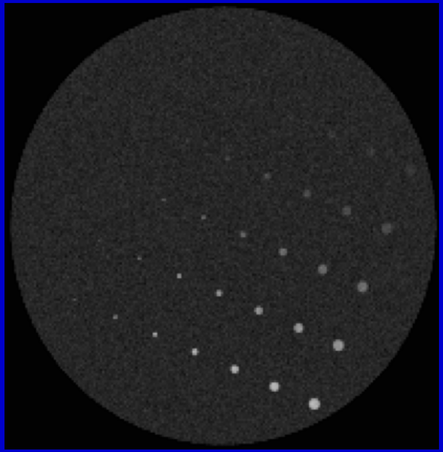
C. Tomographic Images

Simulated photon counting images:





35 keV, Mo anode



35 keV, Mo anode

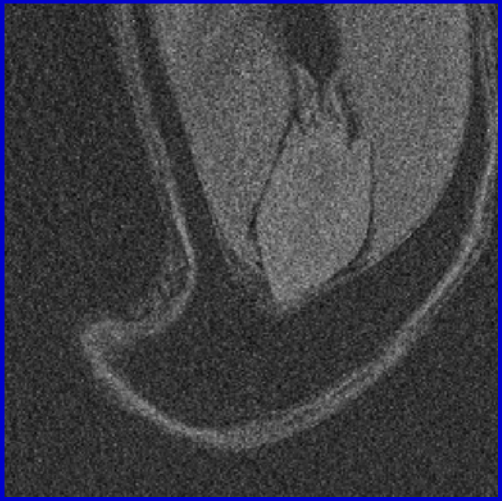
low contrast phantome:
water and fat disks in breast tissue

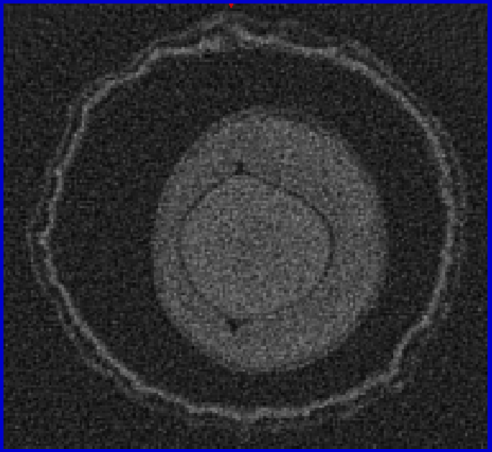
high contrast:
calcium disks in
breast tissue

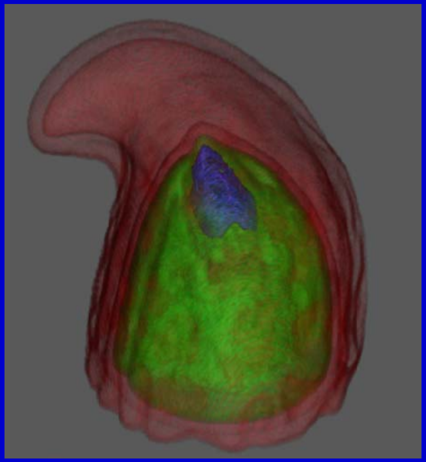
plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

CT image of a peanut
with Medipix 2



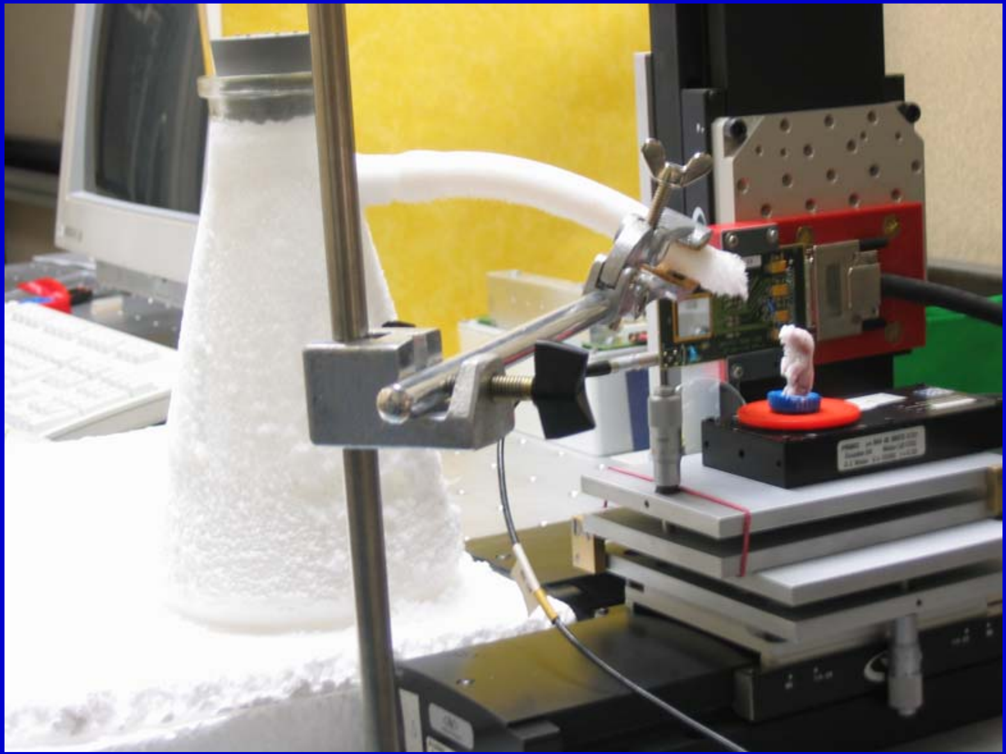




plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

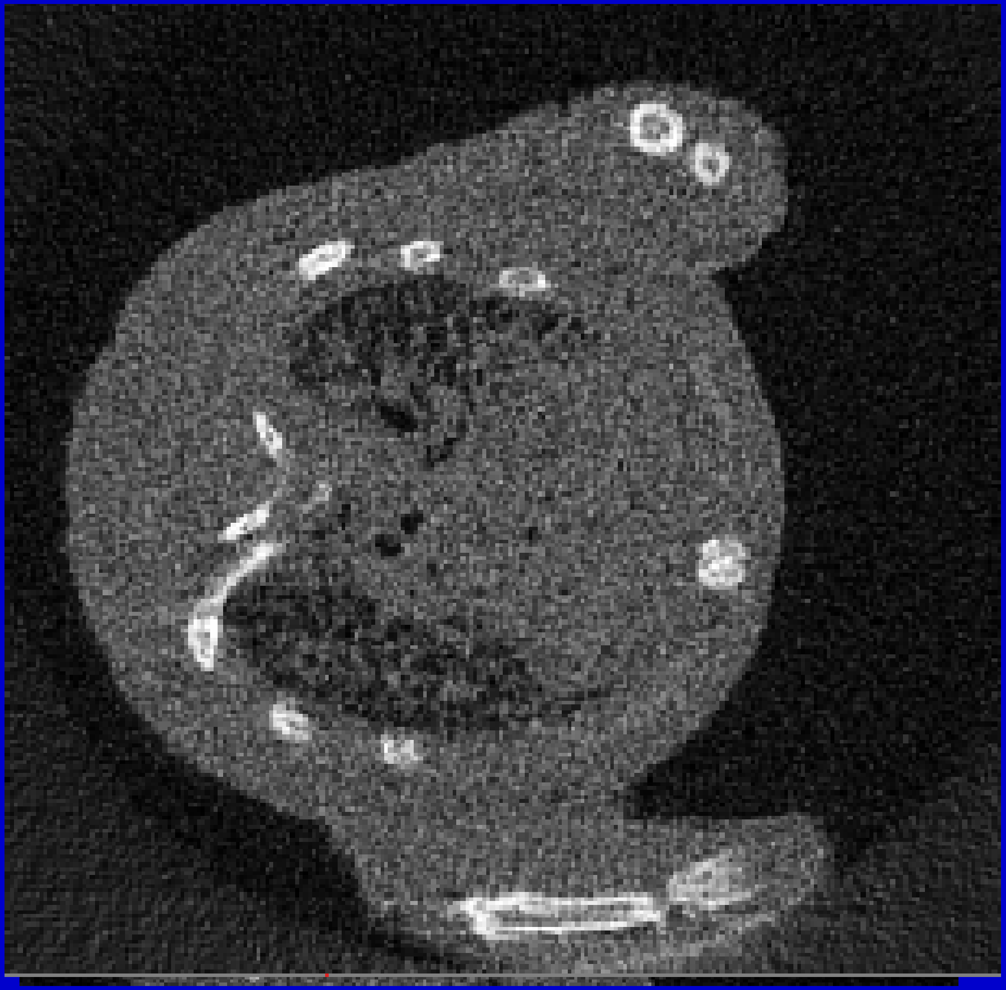
CT image of a mouse with Medipix 2



plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

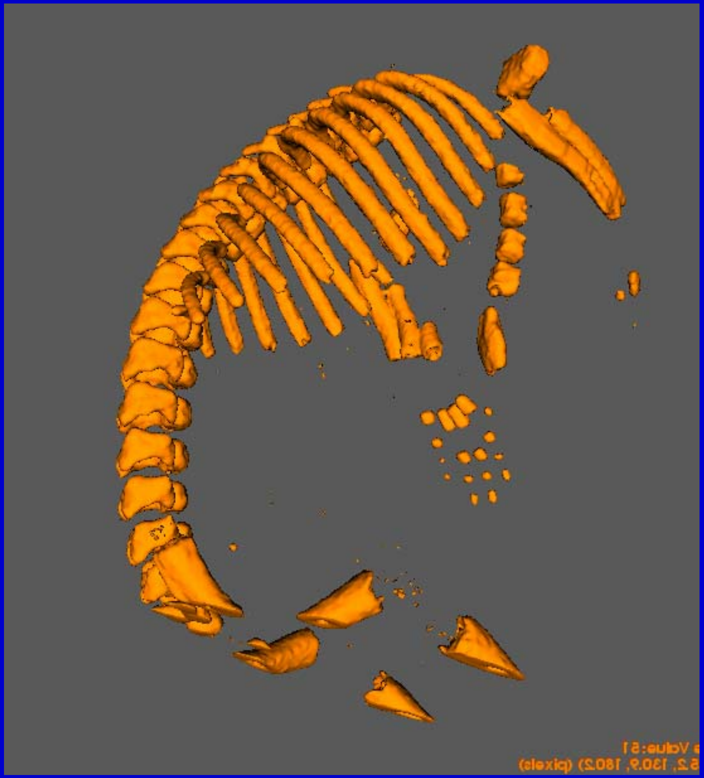
CT image of a mouse
with Medipix 2



plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

CT image of a mouse
with Medipix 2



plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

CT-imaging:

plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

CT-imaging:

plots taken from the work of D.Niederloehner, Erlangen 2005

C. Tomographic Images

Energy sensitive CT-imaging:

$$t_i(E, x, \theta) = -\ln T_i(E, x, \theta) = -\ln \frac{N_i}{N_{io}}(E, x, \theta)$$

$\rightarrow$ $\mu(E, \vec{r})$ $\rightarrow$ $\langle \mu(\vec{r}) \rangle_{opt}$

Many thanks
to my students
for their work

The End

C. Tomographic Images

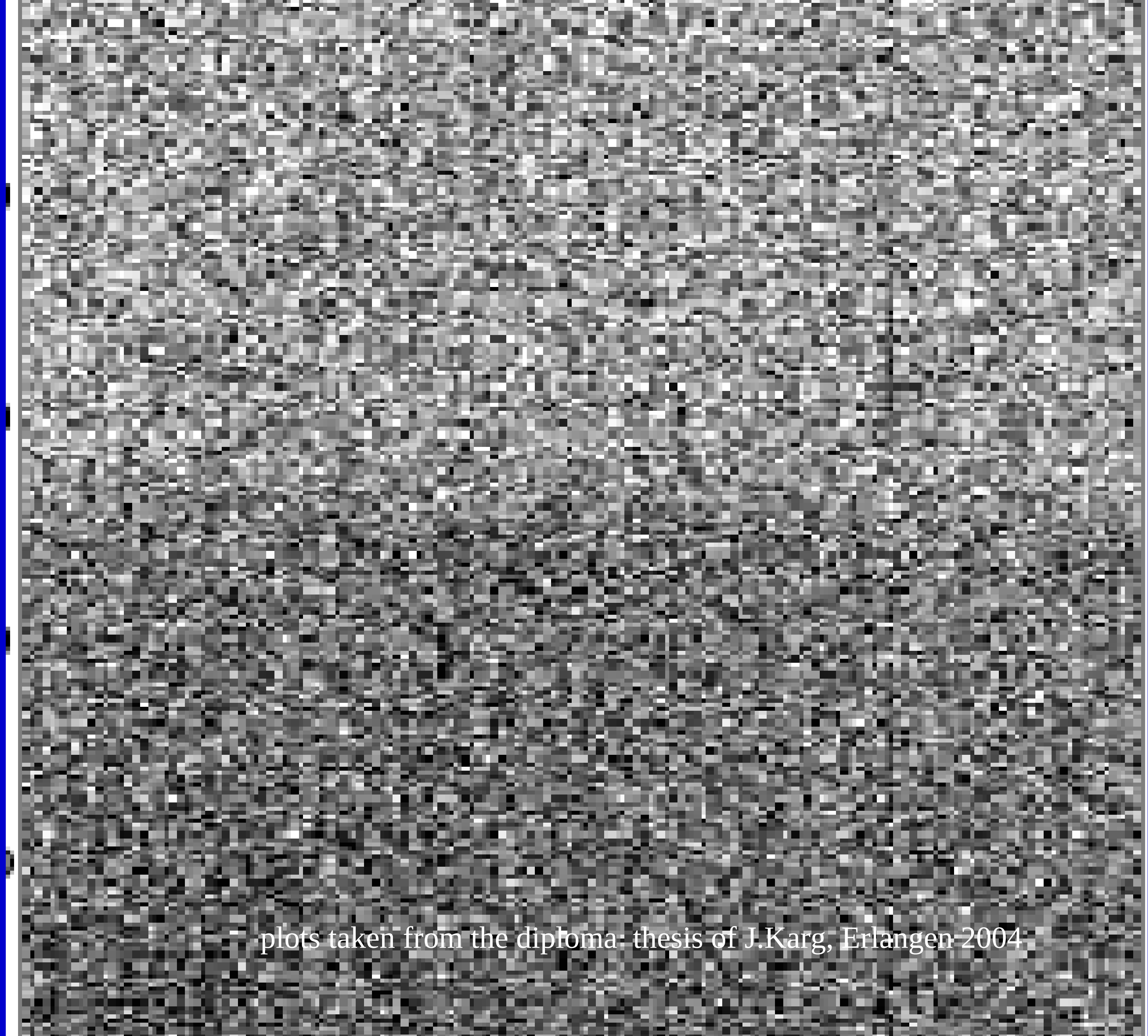
CT-imaging:

1. Take logarithm: $\eta_\theta(t) = \ln \frac{I_0}{I_\theta(t)}$
2. Filter with kernel: $\zeta_\theta(t') = \int_{-\infty}^{\infty} \eta_\theta(t) h(t-t') \mathrm{d}t$
3. Back project: $f(x,y) = \int_0^\pi \zeta_\theta(x\cos\theta + y\sin\theta)\mathrm{d}\theta$

$$t(x,\theta) = -\ln T(x,\theta) = -\ln \frac{N}{N_o}(x,\theta)$$

$$\rightarrow \quad \tilde{\mu}(\vec{r}) = \left\langle \mu(E,\vec{r}) \right\rangle_{Det}$$

plots taken from the diploma thesis of M.Firsching, Erlangen 2005

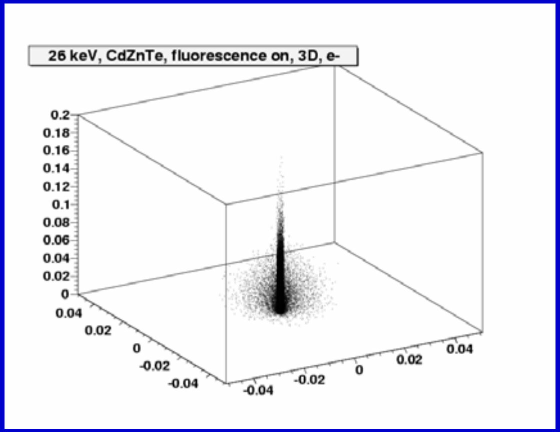


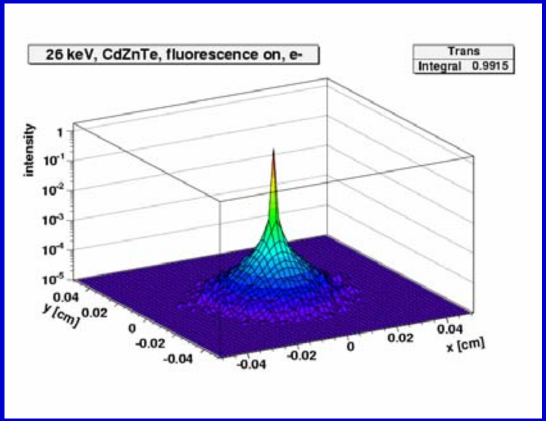
plots taken from the diploma thesis of J.Karg, Erlangen 2004

1) physics of X-Ray interaction with sensor material

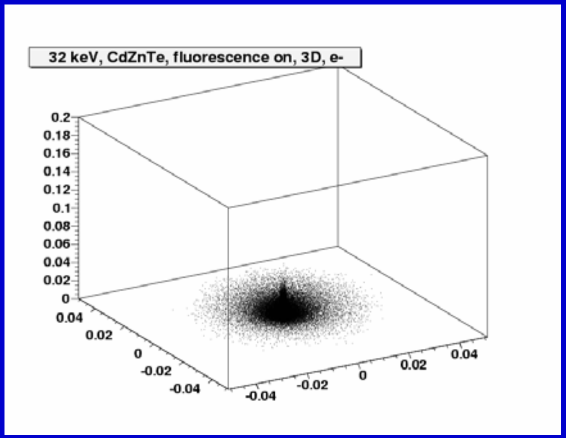
Propagation of secondary photons:

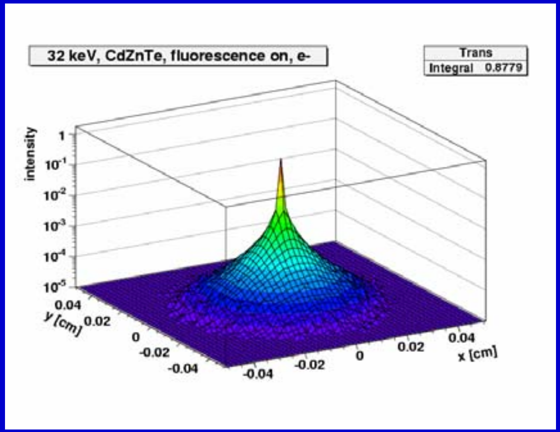
CZT
26 keV





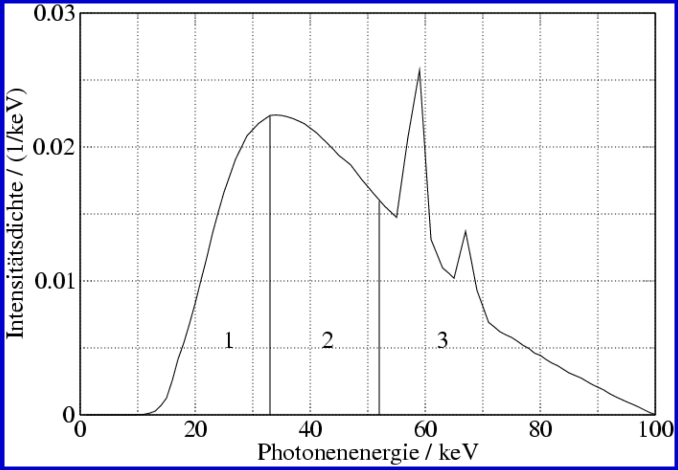
CZT
32 keV

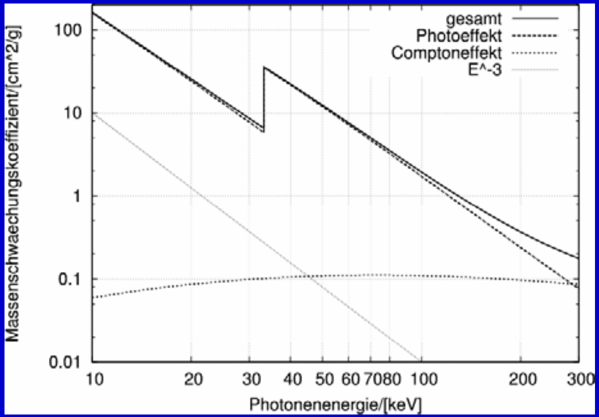


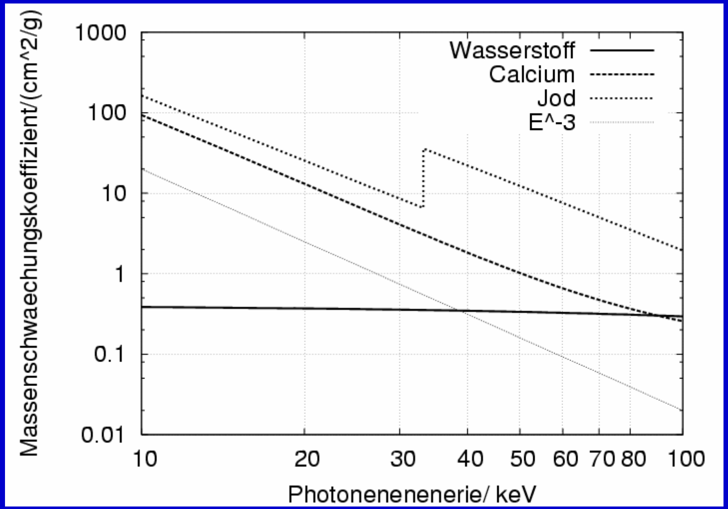


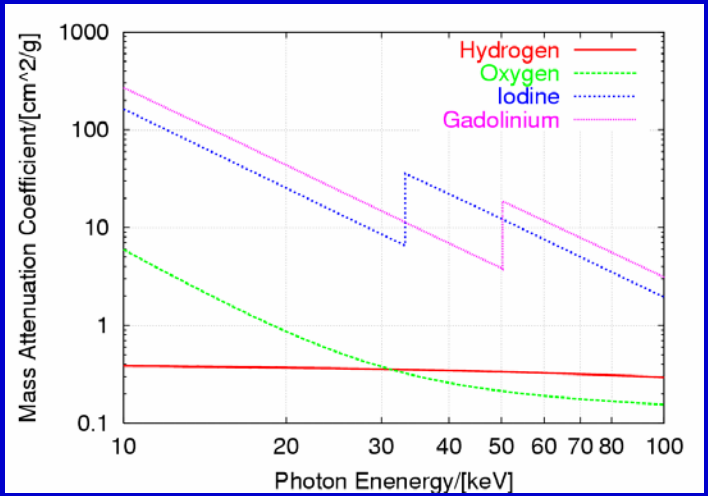
plots taken from the work of J.Durst, Erlangen 2004

Spectral attenuation coefficients









$$\int_{E_{th}}^{E_{max}} \int_{0}^{L} \mu\,(E,\vec{r})\,dl\,dE \;=\; \ln \frac{\int I(E,\vec{r})\,dE}{\int I_o(E)\,dE}$$

ge

$$\int_{0}^{L} \mu\,(E,\vec{r})\,dl \;=\; \ln \frac{I(E,\vec{r})}{I_o(E)}$$

) dl

dE

E

$$\frac{I(x)}{I_o} \;=\; \ln \frac{\int I(E,x)\,dE}{\int I_o(E)\,dE}$$

$$\int_{0}^{L} \mu\,(E,\vec{r})\,dl \;=\; \ln \frac{I(E,\vec{r})}{I_o(E)}$$

$$P(x) \;=\; \ln \frac{I(x)}{I_o} \;=\; \int_{0}^{L} \mu\,(\vec{r})\,dl$$

$$P(x) \;=\; \ln \frac{I(x)}{I_o}$$

$$\mu\,(E,\vec{r})$$

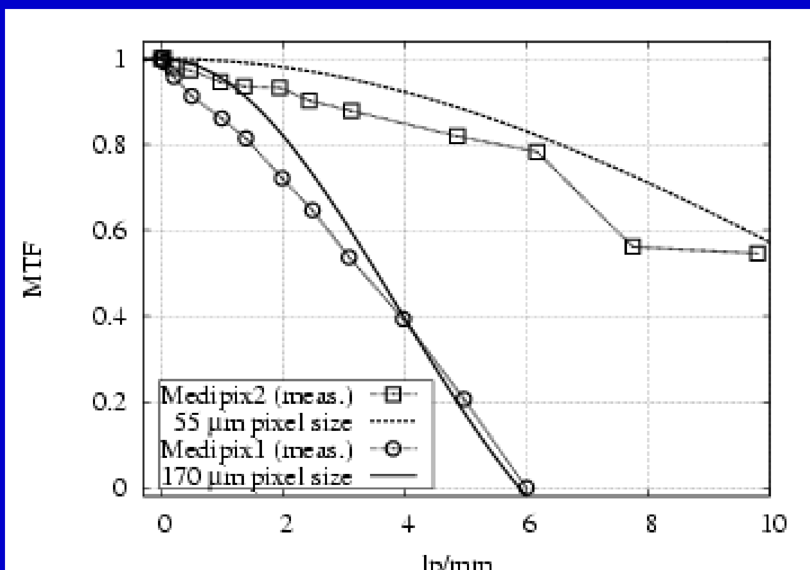
$$\int_{0}^{L} \mu\,(E,\vec{r})\,dl$$

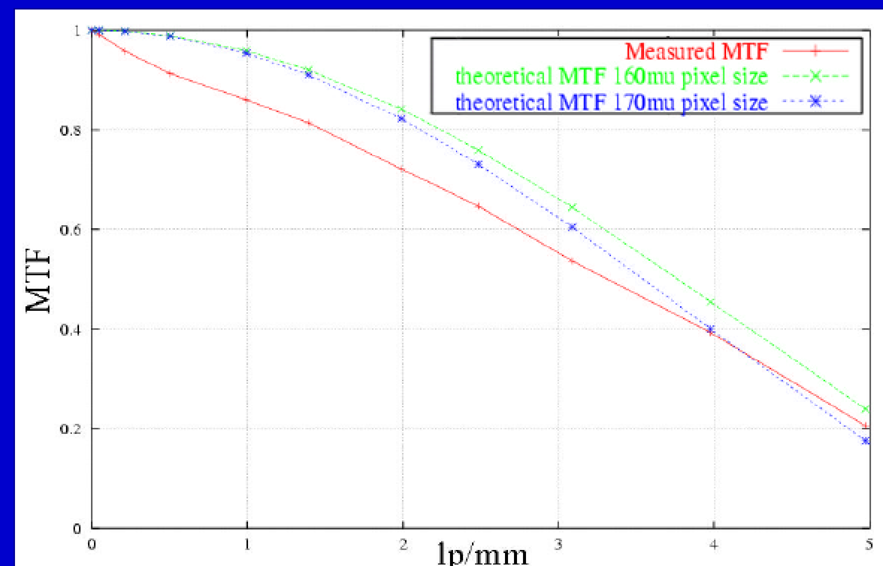
$$\ln \frac{I(E,x)}{I_o(E)} = \frac{\int_{E_{th}}^{E_{max}} I_o(E)\, e^{\int_0^L \mu\,(E,\vec{r})\,dl}\, dE}{\int I_o(E)\,dE} \;=\; \ln \frac{\int I(E,\vec{r})\,dE}{\int I_o(E)\,dE}$$

$$I_o(E)\, e^{\int_0^L \mu\,(E,\vec{r})\,dl} \;=\; I(E)$$

$$I(x) \;=\; I_o\, e^{\int_0^L \mu\,(\vec{r})\,dl}$$

position resolution: MTF with Medipix 1 and 2:





Medipix 1

plots taken from the work of F.Pfeiffer, M.Hoheisel, Erlangen 2004