Theory of Vibrational Properties of Disordered Solids

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We review a theory of vibrational properties of disordered solids [1–2] which is based on the concept of an elastic continuum in which the quenched disorder is assumed to be represented by spatially fluctuating elastic constants. The model is solved by standard field-theoretic techniques leading to an effective Lagrangian which allows for a saddle-point approximation, which is the self-consistent Born approximation (SCBA). The SCBA allows for the calculation of the vibrational density of states $g(\omega)$ as well as the full k-dependent vibrational spectrum $S(k,\omega)$, which can be measured by coherent inelastic neutron or X-ray scattering. This leads to a relation between the excess of $g(\omega)$ over the Debye density of states (boson peak) and the width of the Brillouin peak in $S(k,\omega)$ (sound attenuation constant) [3]. The boson peak can in general be explained as the lower frequency bound of a band of disorder-induced harmonic irregular states. These states are neither propagating nor localized, i.e. they involve a non-vanishing energy diffusion constant $D(\omega)$. $D(\omega)$ can be calculated within the present theory from the Gaussian terms beyond the saddle-point. Together with the inelastic two-level mechanism this leads to an explanation of the characteristic shoulder ("dip") in the temperature dependence of the thermal conductivity of glasses [2]. The dip is caused by the strong increase of the disorder scattering in the boson-peak frequency range.
The theory has been generalized recently to allow for spatial correlations of the elastic-constant fluctuations and anharmonic interactions. The correlations lead to a reinforcement of the boson peak [4]. The anharmonic interaction produces a low-frequency Akhiezer-like sound damping, which increases quadratically with frequency and linearly with temperature, but involves the disorder-induced correlations.
Based on the present model and the assumption of the existence of spatial fluctuations of elasto-optic (Pockels) constants a theory for the anomalous low-frequency Raman spectrum of disordered solids has been formulated. The spectrum turns out not to be proportional to $g(\omega)$ (as was widely believed) but to a weighted q-integral over the longitudinal and transverse excitation spectra [3]. Using this theory one can make consistency checks beetween thermodynamic, Raman, neutron and X-Ray data. Model calculations show good agreement with experiments and simulations.

References

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