High Pressure:

Making it, Measuring it, and Avoiding Pitfalls

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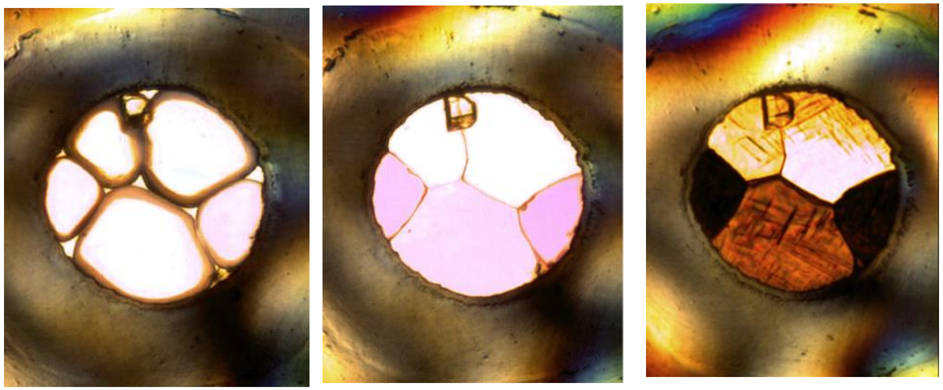
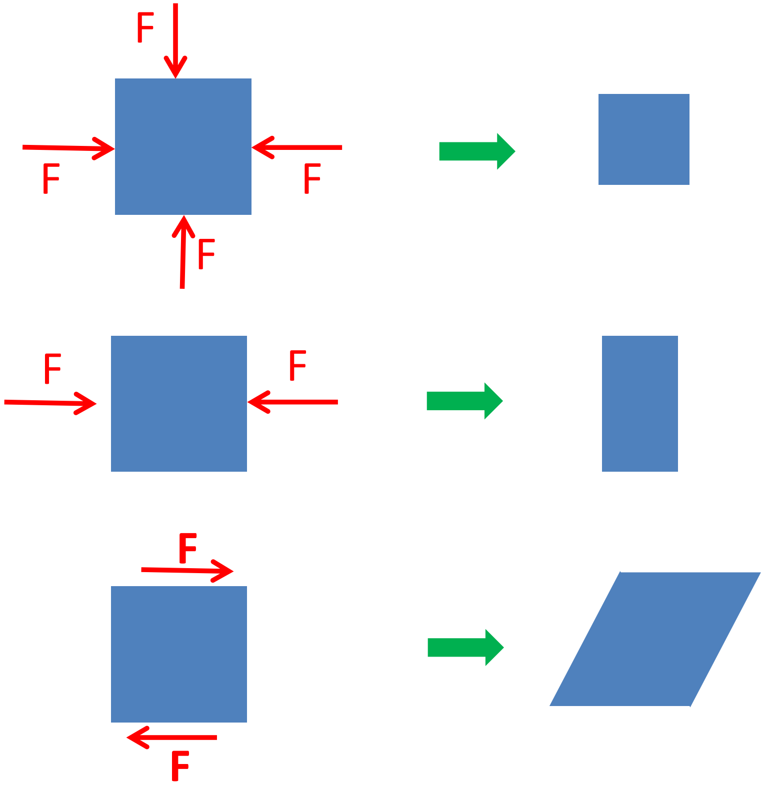
Sorbonne Université, Paris



K. Syassen, MPI Stuttgart

What is pressure?

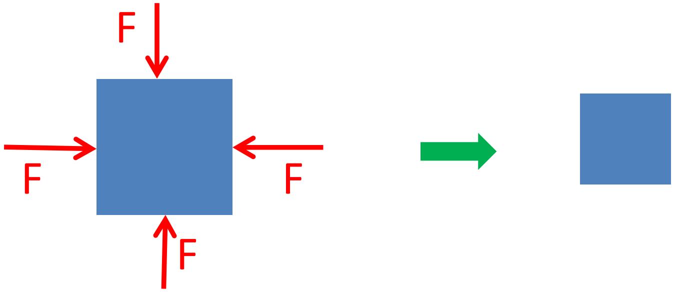
« P = Force/Area »



Conclusion: Pressure can’t be a simple number

Pressure (« Stress ») : A rank-2 tensor

$* = 0$

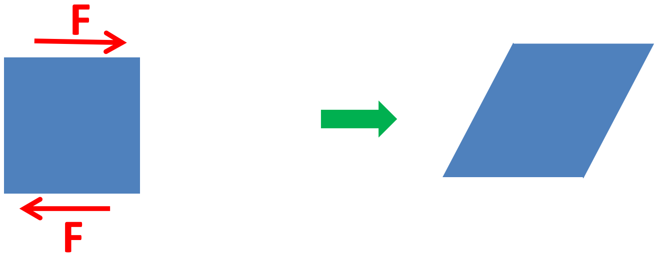


$$\sigma = \begin{pmatrix} -p & & * \\ & -p & \\ * & & -p \end{pmatrix}$$

hydrostatic stress
(« pressure »)

$$\sigma = \begin{pmatrix} \pi & & * \\ & 0 & \\ * & & 0 \end{pmatrix}$$

uniaxial stress
(« pressure »)



$$\sigma = \begin{pmatrix} 0 & 0 & \pi \\ 0 & 0 & 0 \\ \pi & 0 & 0 \end{pmatrix}$$

shear stress
(« pressure »)

Remarks

1. $\sigma(\mathbf{r}) = (\sigma_{ij}(\mathbf{r}))$ → in general dependent on r!
   if not: « homogeneous » pressure

2. Can always define a hydrostatic component: $p = \frac{1}{3} \cdot \mathrm{Tr}\, \sigma$

3. $\sigma_{ij}(\mathbf{r}) = \sigma_{ji}(\mathbf{r})$ → only 6 components
   actually only 3 (« principal stresses »)

Example

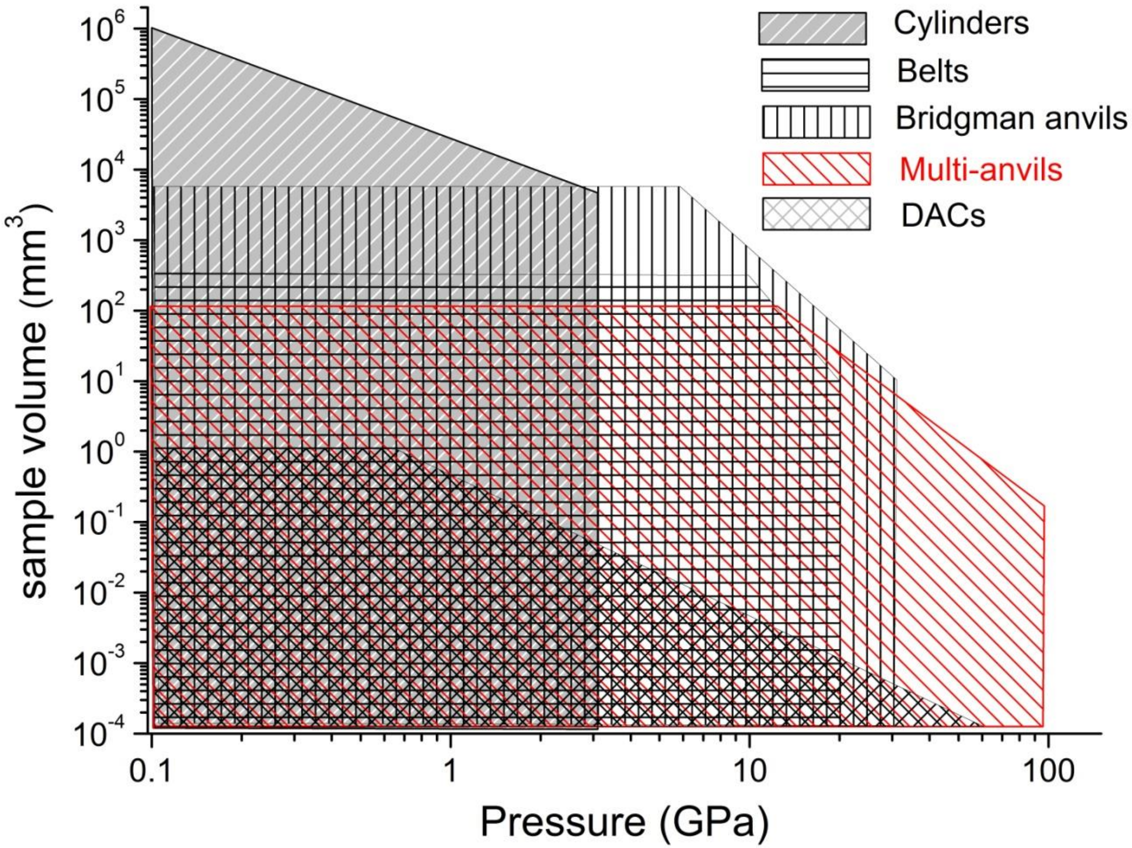
hydrostatic or non-hydrostatic?

$$\sigma = \begin{pmatrix} \pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{pmatrix}$$

$$\pi = \rho\, g\, z$$

→Hydrostatic but not homogeneous

Making pressure



S. Klotz, in: *Techniques in High Pressure Neutron Scattering*, CRC Press, 2013

The cylinder

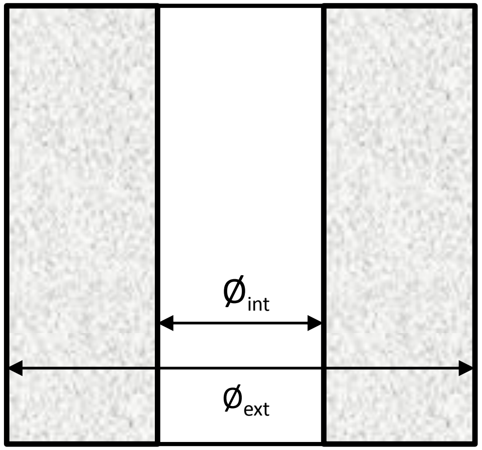
The most common high pressure device

- Pipes
- Car engins
- Hydraulic systems
- High pressure gas bottles
- Pressure cookers
- Autoclaves (= high P/T vessels for synthesis)
- Food processing

- High pressure research equipment

Pressure limit: Useful facts

‘monobloc ‘



$K = Ø_{ext} / Ø_{int}$ ‘wall ratio’

σ_Y: yield strength

Onset of plasticity:

$$P = \sigma_Y \frac{(K^2 - 1)}{\sqrt{3}\, K^2}$$

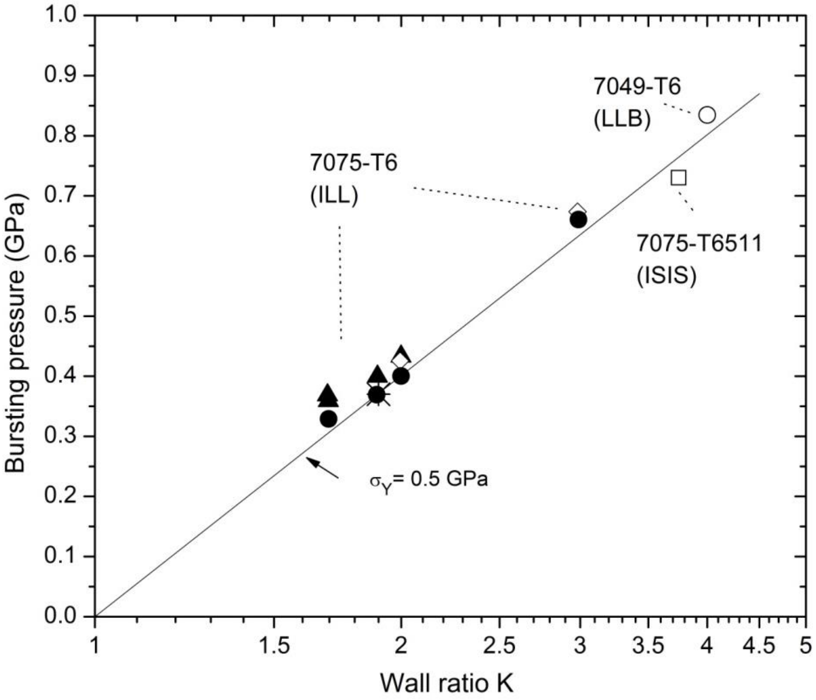
Burst pressure:

$$P = \frac{2\sigma_Y}{\sqrt{3}} \ln K$$

S. Klotz, in: *Techniques in High Pressure Neutron Scattering*, CRC Press, 2013

Burst pressure

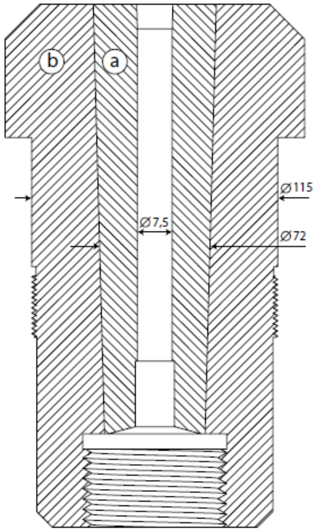
$$P = \frac{2\sigma_Y}{\sqrt{3}} \ln K$$

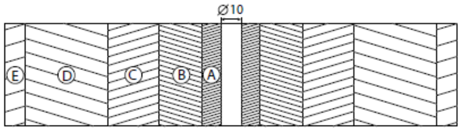


S. Klotz, in: *Techniques in High Pressure Neutron Scattering*, CRC Press, 2013

Frettage

‘compound cylinder‘

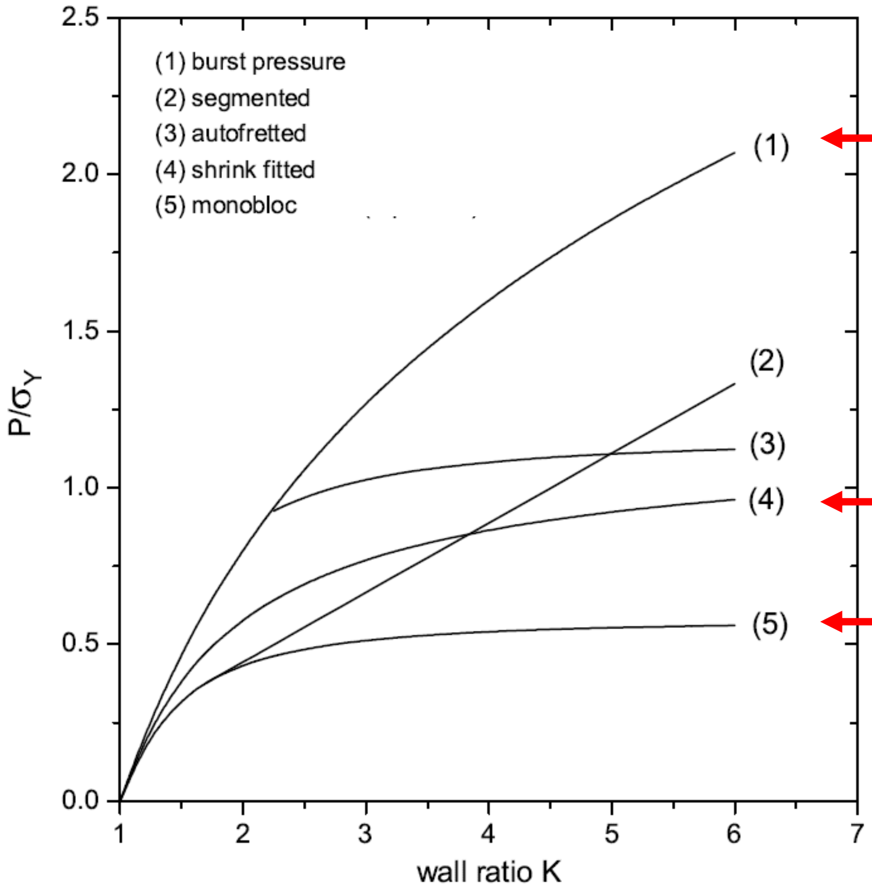




$$P = \frac{2\sigma_Y}{\sqrt{3}} \, \frac{K-1}{K}$$

Monobloc: $$P = \sigma_Y \frac{(K^2-1)}{\sqrt{3}\,K^2}$$

Cylinder: summary



burst P

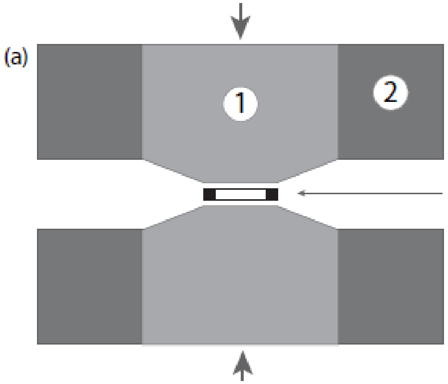
compound

monobloc

S. Klotz, in: *Techniques in High Pressure Neutron Scattering*, CRC Press, 2013

Making pressure: Bridgman anvils

(flat, profiled, toroidal)



Making pressure: The Paris-Edinburgh cell/press

« Compact high-capacity press for HP research applications»

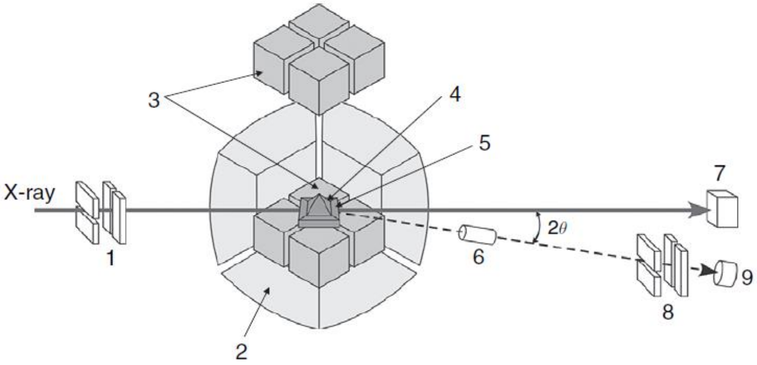
Compact: 10-100 kg
High capacity: 50-450 tonnes
Research: P=1-40 GPa

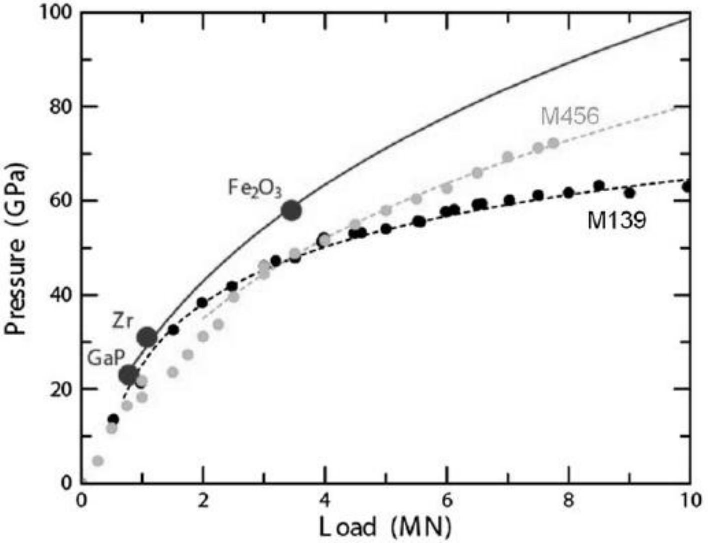
See talks Kono, Le Godec, Perillat, Sanloup

Making pressure: Multianvil cells

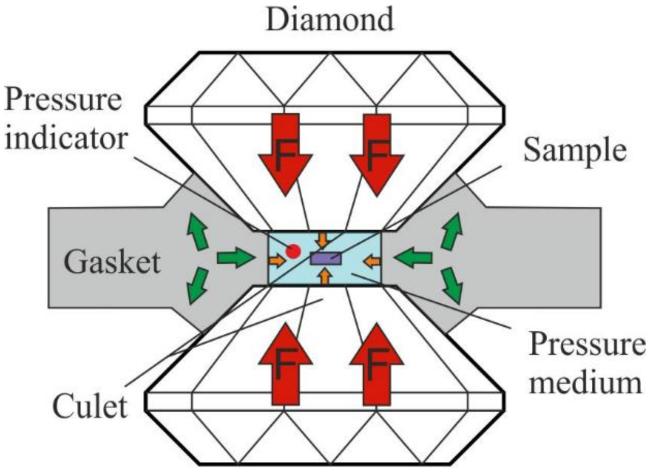
ID06 ESRF, typical: 20 GPa/2000K

E. Ito, Misasa





Making pressure: The diamond anvil cell (DAC)



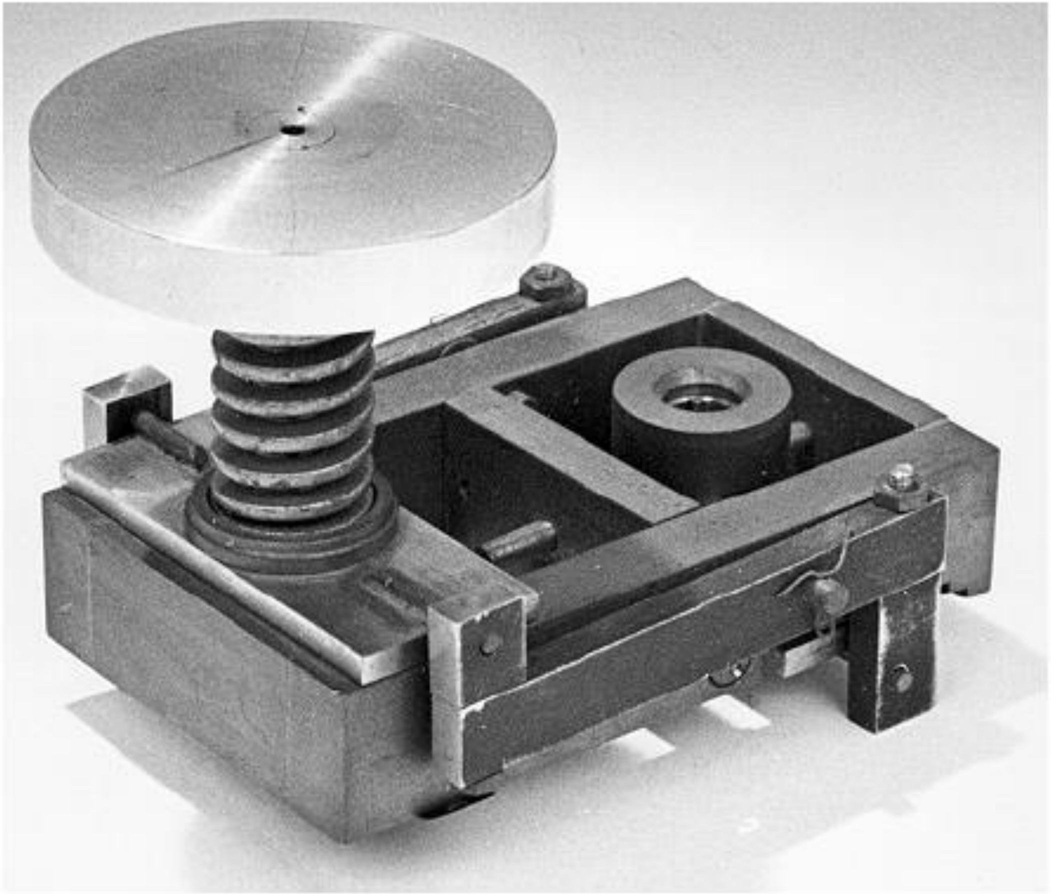
See:
50% of all talks of this School!

Fig. 2. Charles Weir, ca. 1960.

Fig. 3. A. Van Valkenburg, 1962.

NBS Laboratories, Washington, 1958

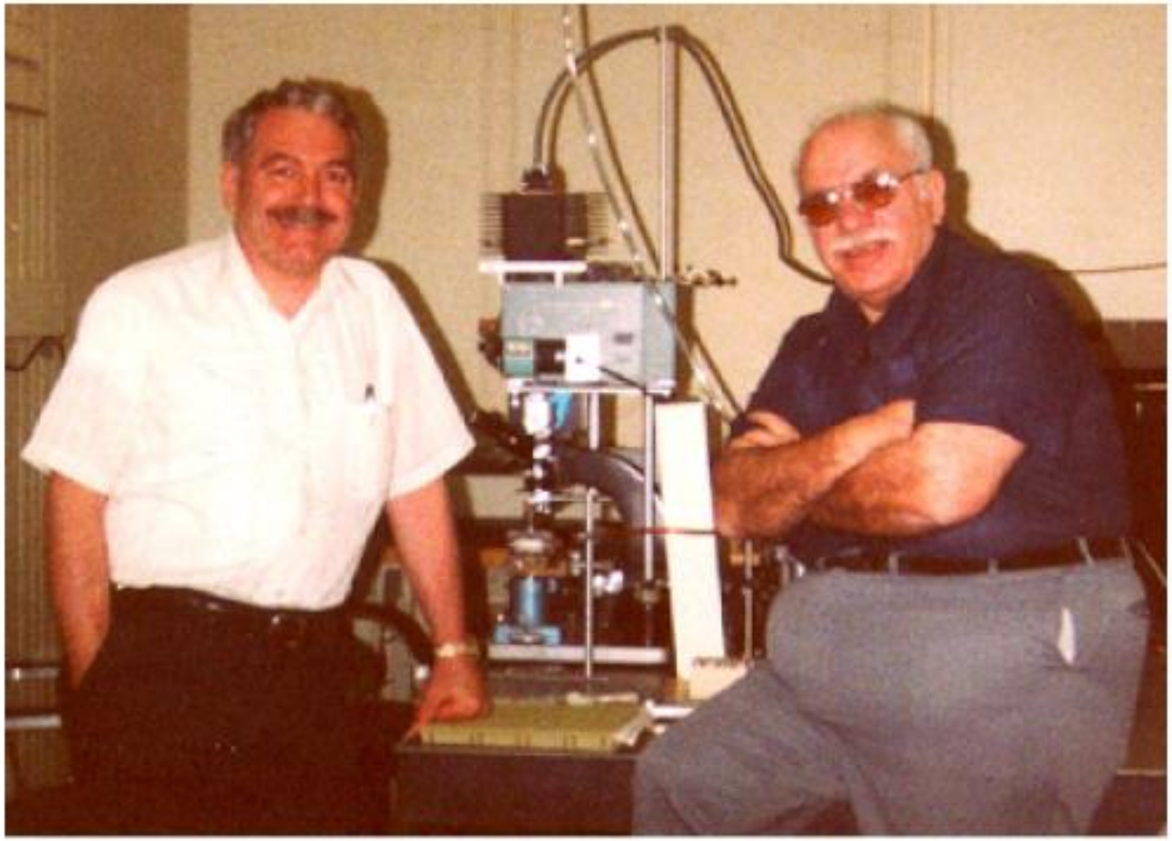
Overview: W. Basset, HPR 29, 163 (2009)



« …Charles E. Weir fabricated it utilizing only a lathe, drill press, hack saw, soldering gun, threading tools, files, and a high speed grinding wheel to polish the culets… »

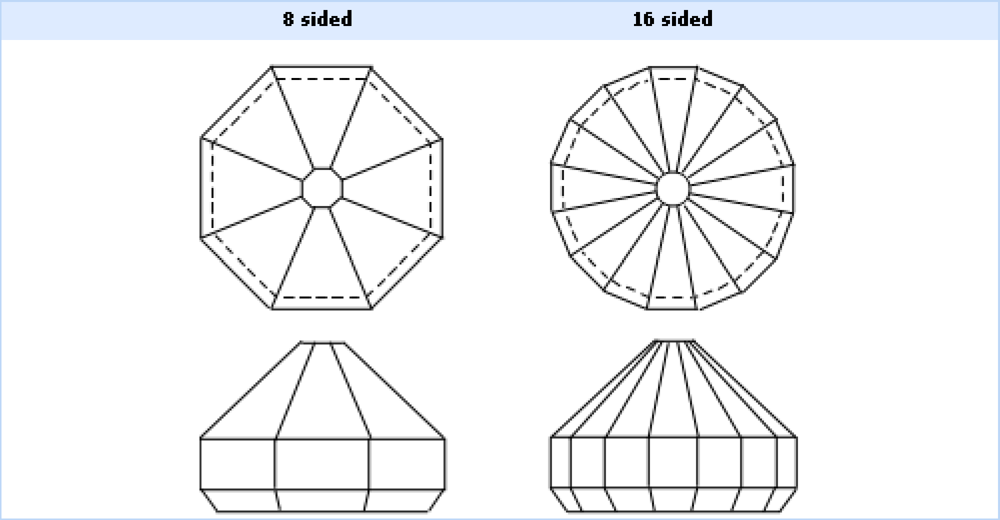
(G. Piermarini & S. Block)

1972: The ruby pressure gauge



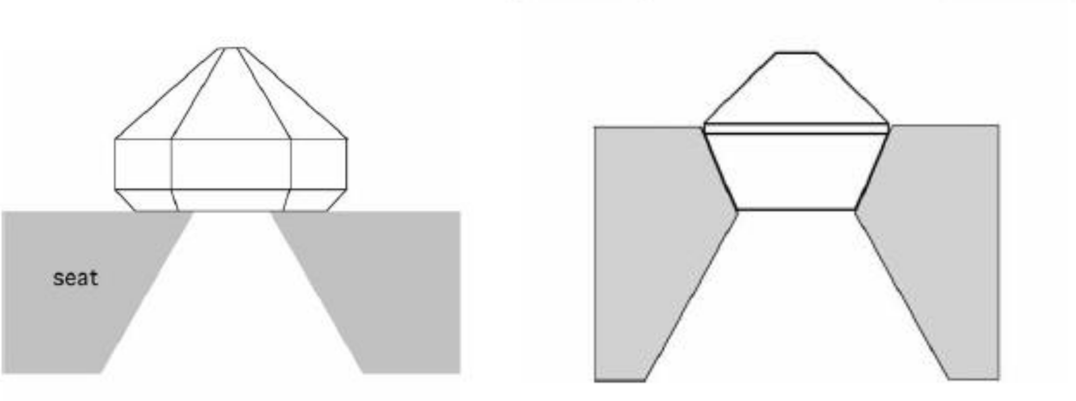
G. Piermarini & S. Block, ca. 1984

What makes a DAC:



1. Diamonds

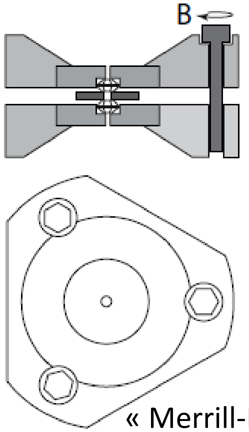
(see following talk Hantsetters)



1. Seats

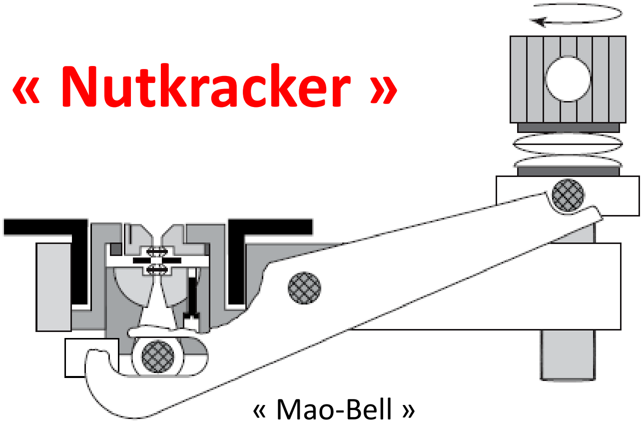
3. Load frames

Screws



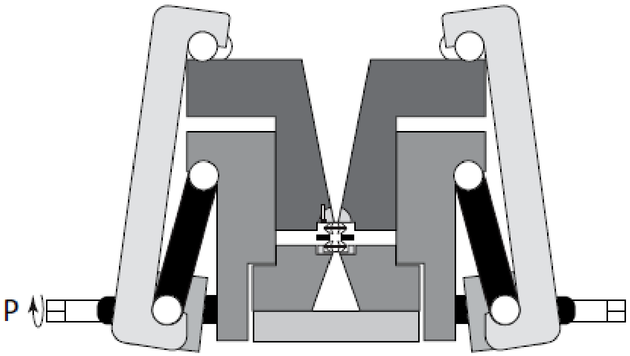
« Merrill-Bassett »

« Nutkracker »



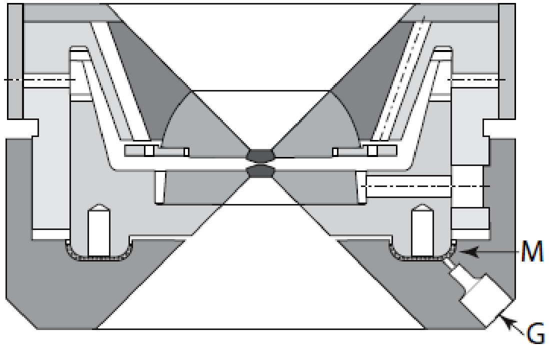
« Mao-Bell »

Toggle-latch



« Syassen-Holzapfel »

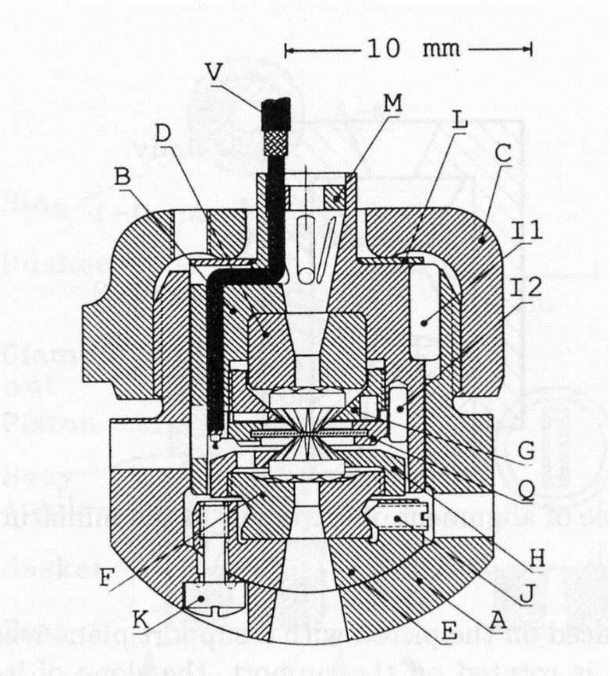
Membrane



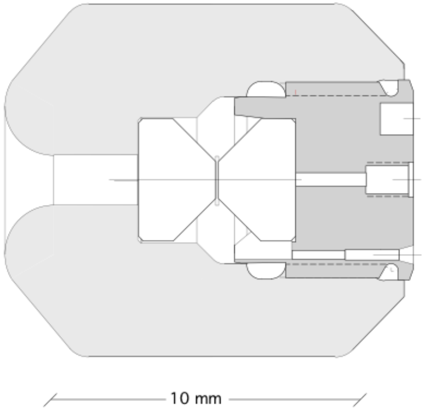
S. Klotz, in: *High Pressure Physics*, CRC Press, 2010

Miniature DACs

for high magnetic fields



S. Tozer et al.
RSI 64, 2607 (1993)



D.E. Graf
HPR 2011

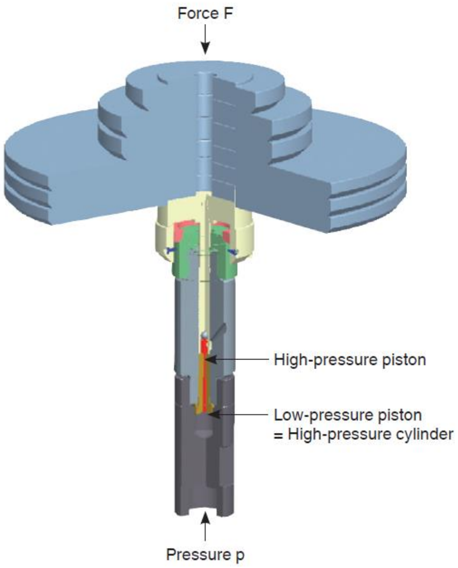
All plastic diamond anvil cell, ~30 kbar

Measuring pressure

Primary pressure scales:

(→use only the definition of pressure)

$P = F/A$



High pressure balances

P-range: 0-5 kbar
Accuracy: 0.02%

High pressure balances to 3 GPa

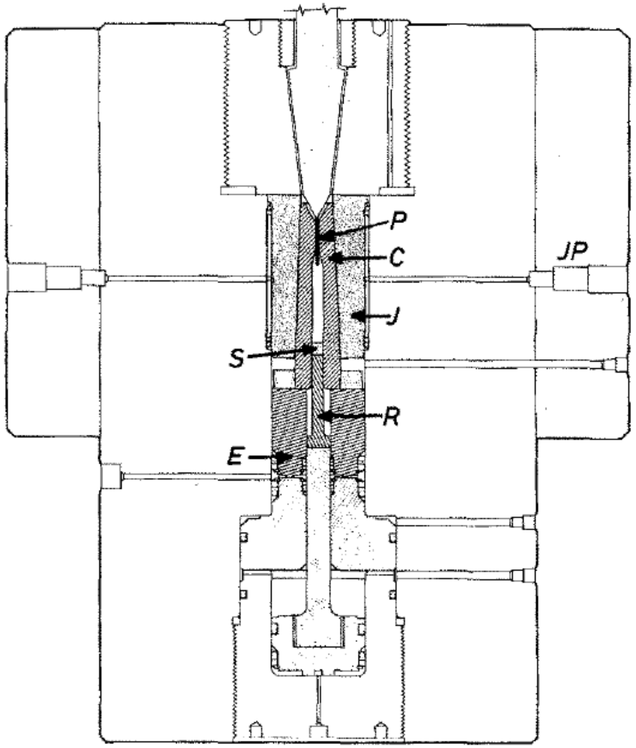


FIG. 1. 26-kbar piston gauge, schematic: P, carboloy gauge piston; C, cylinder, J. jacket; JP, connection for jacket pressure; S, seal; R, pressure generator ram; E, end load ram.

Heydemann, J. Appl. Phys. (1967)

- Technically difficult
- Not commercially available
- High exploitation costs

Accuracy: ~ 0.1%

Secondary pressure scales

= Methods which:

- Are more adapted to a specific P range and device
- Have been calibrated to a primary standard

- Volume scales: $V=V(P)$
- Optical scales: $\lambda=\lambda(P)$

The Decker NaCl scale (1971)

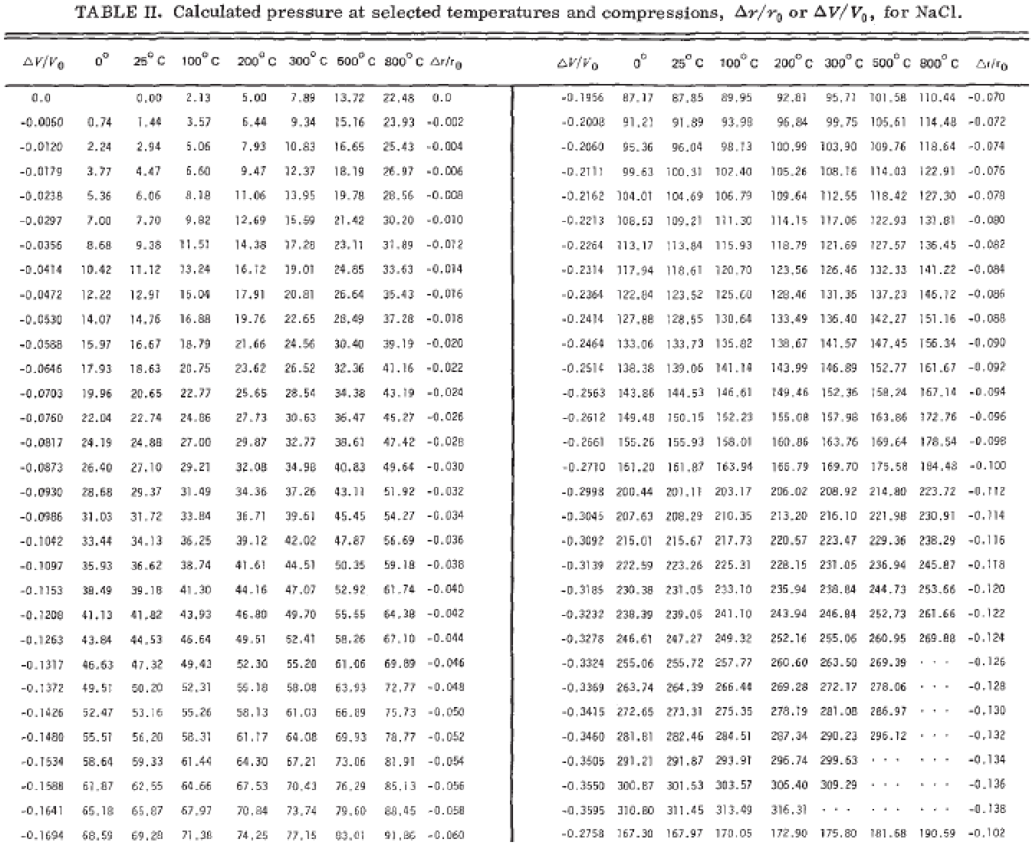


TABLE II. Calculated pressure at selected temperatures and compressions, $\Delta r/r_0$ or $\Delta V/V_0$, for NaCl.

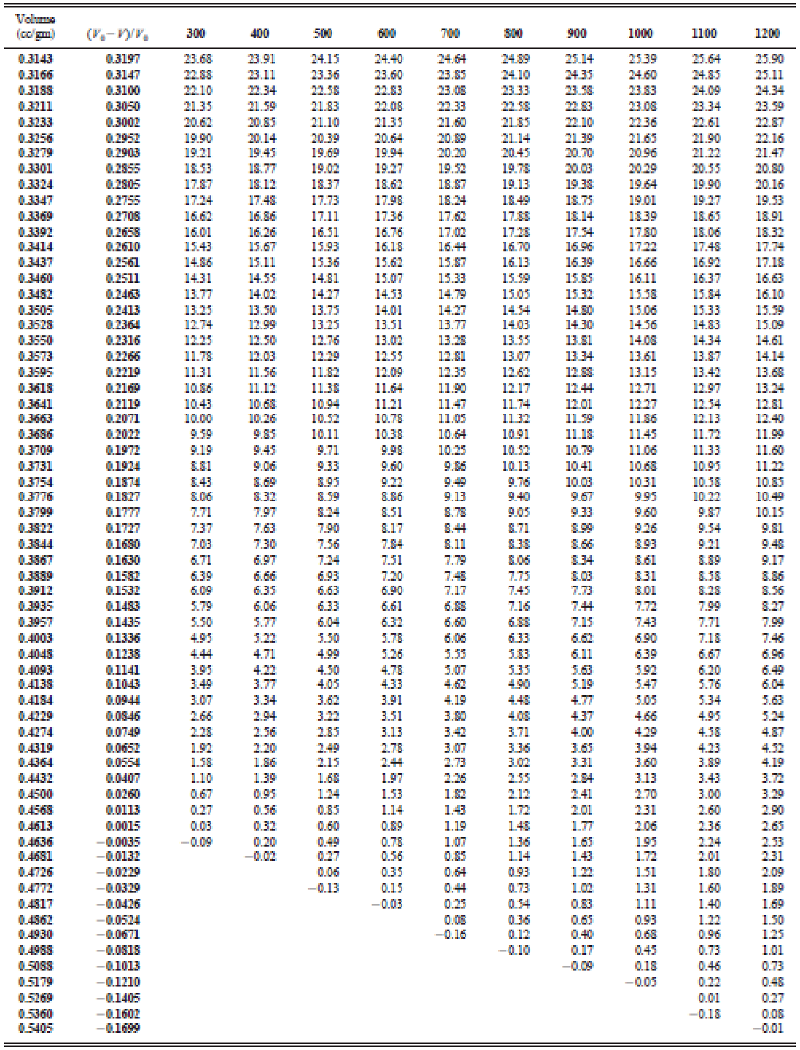
$\Delta V/V_0$	0°	25° C	100° C	200° C	300° C	600° C	800° C	$\Delta r/r_0$	$\Delta V/V_0$	0°	25° C	100° C	200° C	300° C	500° C	800° C	$\Delta r/r_0$
0.0		0.00	2.13	5.00	7.89	13.72	22.48	0.0	-0.1956	87.17	87.85	89.95	92.81	95.71	101.58	110.44	-0.070
-0.0060	0.74	1.44	3.57	6.44	9.34	15.16	23.93	-0.002	-0.2008	91.21	91.89	93.98	96.84	99.75	105.61	114.48	-0.072
-0.0120	2.24	2.94	5.06	7.93	10.83	16.65	25.43	-0.004	-0.2060	95.36	96.04	98.13	100.99	103.90	109.76	118.64	-0.074
-0.0179	3.77	4.47	6.60	9.47	12.37	18.19	26.97	-0.006	-0.2111	99.63	100.31	102.40	105.26	108.16	114.03	122.91	-0.076
-0.0238	5.36	6.06	8.18	11.06	13.95	19.78	28.56	-0.008	-0.2162	104.01	104.69	106.79	109.64	112.55	118.42	127.30	-0.078
-0.0297	7.00	7.70	9.82	12.69	15.59	21.42	30.20	-0.010	-0.2213	108.53	109.21	111.30	114.15	117.06	122.93	131.81	-0.080
-0.0356	8.68	9.38	11.51	14.38	17.28	23.11	31.89	-0.012	-0.2264	113.17	113.84	115.93	118.79	121.69	127.57	136.45	-0.082
-0.0414	10.42	11.12	13.24	16.12	19.01	24.85	33.63	-0.014	-0.2314	117.94	118.61	120.70	123.56	126.46	132.33	141.22	-0.084
-0.0472	12.22	12.91	15.04	17.91	20.81	26.64	35.43	-0.016	-0.2364	122.84	123.52	125.60	128.46	131.35	137.23	146.12	-0.086
-0.0530	14.07	14.76	16.88	19.76	22.65	28.49	37.28	-0.018	-0.2414	127.88	128.55	130.64	133.49	136.40	142.27	151.16	-0.088
-0.0588	15.97	16.67	18.79	21.66	24.56	30.40	39.19	-0.020	-0.2464	133.06	133.73	135.82	138.67	141.57	147.45	156.34	-0.090
-0.0646	17.93	18.63	20.75	23.62	26.52	32.36	41.16	-0.022	-0.2514	138.38	139.06	141.14	143.99	146.89	152.77	161.67	-0.092
-0.0703	19.96	20.65	22.77	25.65	28.54	34.38	43.19	-0.024	-0.2563	143.86	144.53	146.61	149.46	152.36	158.24	167.14	-0.094
-0.0760	22.04	22.74	24.86	27.73	30.63	36.47	45.27	-0.026	-0.2612	149.48	150.15	152.23	155.08	157.98	163.86	172.76	-0.096
-0.0817	24.19	24.88	27.00	29.87	32.77	38.61	47.42	-0.028	-0.2661	155.26	155.93	158.01	160.86	163.76	169.64	178.54	-0.098
-0.0873	26.40	27.10	29.21	32.08	34.98	40.83	49.64	-0.030	-0.2710	161.20	161.87	163.94	166.79	169.70	175.58	184.48	-0.100
-0.0930	28.68	29.37	31.49	34.36	37.26	43.11	51.92	-0.032	-0.2998	200.44	201.11	203.17	206.02	208.92	214.80	223.72	-0.112
-0.0986	31.03	31.72	33.84	36.71	39.61	45.45	54.27	-0.034	-0.3045	207.63	208.29	210.35	213.20	216.10	221.98	230.91	-0.114
-0.1042	33.44	34.13	36.25	39.12	42.02	47.87	56.69	-0.036	-0.3092	215.01	215.67	217.73	220.57	223.47	229.36	238.29	-0.116
-0.1097	35.93	36.62	38.74	41.61	44.51	50.35	59.18	-0.038	-0.3139	222.59	223.26	225.31	228.15	231.05	236.94	245.87	-0.118
-0.1153	38.49	39.18	41.30	44.16	47.07	52.92	61.74	-0.040	-0.3185	230.38	231.05	233.10	235.94	238.84	244.73	253.66	-0.120
-0.1208	41.13	41.82	43.93	46.80	49.70	55.55	64.38	-0.042	-0.3232	238.39	239.05	241.10	243.94	246.84	252.73	261.66	-0.122
-0.1263	43.84	44.53	46.64	49.51	52.41	58.26	67.10	-0.044	-0.3278	246.61	247.27	249.32	252.16	255.06	260.95	269.88	-0.124
-0.1317	46.63	47.32	49.43	52.30	55.20	61.06	69.89	-0.046	-0.3324	255.06	255.72	257.77	260.60	263.50	269.39	. . .	-0.126
-0.1372	49.51	50.20	52.31	55.18	58.08	63.93	72.77	-0.048	-0.3369	263.74	264.39	266.44	269.28	272.17	278.06	. . .	-0.128
-0.1426	52.47	53.16	55.26	58.13	61.03	66.89	75.73	-0.050	-0.3415	272.65	273.31	275.35	278.19	281.08	286.97	. . .	-0.130
-0.1480	55.51	56.20	58.31	61.17	64.08	69.93	78.77	-0.052	-0.3460	281.81	282.46	284.51	287.34	290.23	296.12	. . .	-0.132
-0.1534	58.64	59.33	61.44	64.30	67.21	73.06	81.91	-0.054	-0.3505	291.21	291.87	293.91	296.74	299.63	. . .	. . .	-0.134
-0.1588	61.87	62.55	64.66	67.53	70.43	76.29	85.13	-0.056	-0.3550	300.87	301.53	303.57	305.40	309.29	. . .	. . .	-0.136
-0.1641	65.18	65.87	67.97	70.84	73.74	79.60	88.45	-0.058	-0.3595	310.80	311.45	313.49	316.31	. . .	. . .	. . .	-0.138
-0.1694	68.59	69.29	71.38	74.25	77.15	83.01	91.86	-0.060	-0.2758	167.30	167.97	170.05	172.90	175.80	181.68	190.59	-0.102

- **“Table”, semi-empirical (interatom. pot. + exp. input params)**
- **P-range: 0-300 kbar**
- **Accuracy: 0-5 % (!?)**

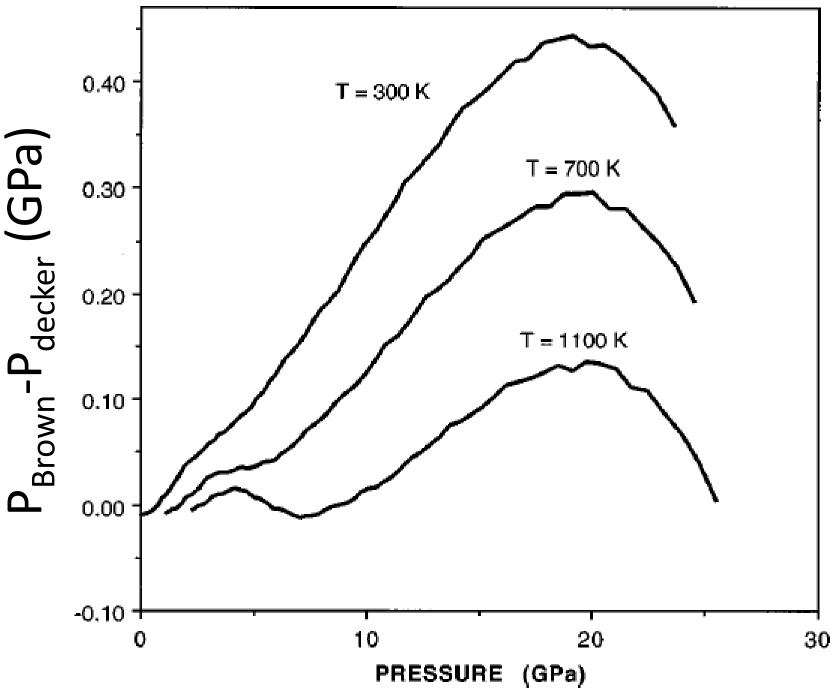
Decker, J. Appl. Phys. 42, 3239 (1971)

Brown’s 1999 NaCl scale

M. Brown, J. Appl. Phys. , 1999



Volume (cc/gm)	$(V_0-V)/V_0$	300	400	500	600	700	800	900	1000	1100	1200
0.3143	0.3197	23.68	23.91	24.15	24.40	24.64	24.89	25.14	25.39	25.64	25.90
0.3166	0.3147	22.88	23.11	23.36	23.60	23.85	24.10	24.35	24.60	24.85	25.11
0.3188	0.3100	22.10	22.34	22.58	22.83	23.08	23.33	23.58	23.83	24.09	24.34
0.3211	0.3050	21.35	21.59	21.83	22.08	22.33	22.58	22.83	23.08	23.34	23.59
0.3233	0.3002	20.62	20.85	21.10	21.35	21.60	21.85	22.10	22.36	22.61	22.87
0.3256	0.2952	19.90	20.14	20.39	20.64	20.89	21.14	21.39	21.65	21.90	22.16
0.3279	0.2903	19.21	19.45	19.69	19.94	20.20	20.45	20.70	20.96	21.22	21.47
0.3301	0.2855	18.53	18.77	19.02	19.27	19.52	19.78	20.03	20.29	20.55	20.80
0.3324	0.2805	17.87	18.12	18.37	18.62	18.87	19.13	19.38	19.64	19.90	20.16
0.3347	0.2755	17.24	17.48	17.73	17.98	18.24	18.49	18.75	19.01	19.27	19.53
0.3369	0.2708	16.62	16.86	17.11	17.36	17.62	17.88	18.14	18.39	18.65	18.91
0.3392	0.2658	16.01	16.26	16.51	16.76	17.02	17.28	17.54	17.80	18.06	18.32
0.3414	0.2610	15.43	15.67	15.93	16.18	16.44	16.70	16.96	17.22	17.48	17.74
0.3437	0.2561	14.86	15.11	15.36	15.62	15.87	16.13	16.39	16.66	16.92	17.18
0.3460	0.2511	14.31	14.55	14.81	15.07	15.33	15.59	15.85	16.11	16.37	16.63
0.3482	0.2463	13.77	14.02	14.27	14.53	14.79	15.05	15.32	15.58	15.84	16.10
0.3505	0.2413	13.25	13.50	13.75	14.01	14.27	14.54	14.80	15.06	15.33	15.59
0.3528	0.2364	12.74	12.99	13.25	13.51	13.77	14.03	14.30	14.56	14.83	15.09
0.3550	0.2316	12.25	12.50	12.76	13.02	13.28	13.55	13.81	14.08	14.34	14.61
0.3573	0.2266	11.78	12.03	12.29	12.55	12.81	13.07	13.34	13.61	13.87	14.14
0.3595	0.2219	11.31	11.56	11.82	12.09	12.35	12.62	12.88	13.15	13.42	13.68
0.3618	0.2169	10.86	11.12	11.38	11.64	11.90	12.17	12.44	12.71	12.97	13.24
0.3641	0.2119	10.43	10.68	10.94	11.21	11.47	11.74	12.01	12.27	12.54	12.81
0.3663	0.2071	10.00	10.26	10.52	10.78	11.05	11.32	11.59	11.86	12.13	12.40
0.3686	0.2022	9.59	9.85	10.11	10.38	10.64	10.91	11.18	11.45	11.72	11.99
0.3709	0.1972	9.19	9.45	9.71	9.98	10.25	10.52	10.79	11.06	11.33	11.60
0.3731	0.1924	8.81	9.06	9.33	9.60	9.86	10.13	10.41	10.68	10.95	11.22
0.3754	0.1874	8.43	8.69	8.95	9.22	9.49	9.76	10.03	10.31	10.58	10.85
0.3776	0.1827	8.06	8.32	8.59	8.86	9.13	9.40	9.67	9.95	10.22	10.49
0.3799	0.1777	7.71	7.97	8.24	8.51	8.78	9.05	9.33	9.60	9.87	10.15
0.3822	0.1727	7.37	7.63	7.90	8.17	8.44	8.71	8.99	9.26	9.54	9.81
0.3844	0.1680	7.03	7.30	7.56	7.84	8.11	8.38	8.66	8.93	9.21	9.48
0.3867	0.1630	6.71	6.97	7.24	7.51	7.79	8.06	8.34	8.61	8.89	9.17
0.3889	0.1582	6.39	6.66	6.93	7.20	7.48	7.75	8.03	8.31	8.58	8.86
0.3912	0.1532	6.09	6.35	6.63	6.90	7.17	7.45	7.73	8.01	8.28	8.56
0.3935	0.1483	5.79	6.06	6.33	6.61	6.88	7.16	7.44	7.72	7.99	8.27
0.3957	0.1435	5.50	5.77	6.04	6.32	6.60	6.88	7.15	7.43	7.71	7.99
0.4003	0.1336	4.95	5.22	5.50	5.78	6.06	6.33	6.62	6.90	7.18	7.46
0.4048	0.1238	4.44	4.71	4.99	5.26	5.55	5.83	6.11	6.39	6.67	6.96
0.4093	0.1141	3.95	4.22	4.50	4.78	5.07	5.35	5.63	5.92	6.20	6.49
0.4138	0.1043	3.49	3.77	4.05	4.33	4.62	4.90	5.19	5.47	5.76	6.04
0.4184	0.0944	3.07	3.34	3.62	3.91	4.19	4.48	4.77	5.05	5.34	5.63
0.4229	0.0846	2.66	2.94	3.22	3.51	3.80	4.08	4.37	4.66	4.95	5.24
0.4274	0.0749	2.28	2.56	2.85	3.13	3.42	3.71	4.00	4.29	4.58	4.87
0.4319	0.0652	1.92	2.20	2.49	2.78	3.07	3.36	3.65	3.94	4.23	4.52
0.4364	0.0554	1.58	1.86	2.15	2.44	2.73	3.02	3.31	3.60	3.89	4.19
0.4432	0.0407	1.10	1.39	1.68	1.97	2.26	2.55	2.84	3.13	3.43	3.72
0.4500	0.0260	0.67	0.95	1.24	1.53	1.82	2.12	2.41	2.70	3.00	3.29
0.4568	0.0113	0.27	0.56	0.85	1.14	1.43	1.72	2.01	2.31	2.60	2.90
0.4613	0.0015	0.03	0.32	0.60	0.89	1.19	1.48	1.77	2.06	2.36	2.65
0.4636	−0.0035	−0.09	0.20	0.49	0.78	1.07	1.36	1.65	1.95	2.24	2.53
0.4681	−0.0132		−0.02	0.27	0.56	0.85	1.14	1.43	1.72	2.01	2.31
0.4726	−0.0229			0.06	0.35	0.64	0.93	1.22	1.51	1.80	2.09
0.4772	−0.0329			−0.13	0.15	0.44	0.73	1.02	1.31	1.60	1.89
0.4817	−0.0426				−0.03	0.25	0.54	0.83	1.11	1.40	1.69
0.4862	−0.0524					0.08	0.36	0.65	0.93	1.22	1.50
0.4930	−0.0671					−0.16	0.12	0.40	0.68	0.96	1.25
0.4988	−0.0818						−0.10	0.17	0.45	0.73	1.01
0.5088	−0.1013							−0.09	0.18	0.46	0.73
0.5179	−0.1210								−0.05	0.22	0.48
0.5269	−0.1405									0.01	0.27
0.5360	−0.1602									−0.18	0.08
0.5405	−0.1699										−0.01



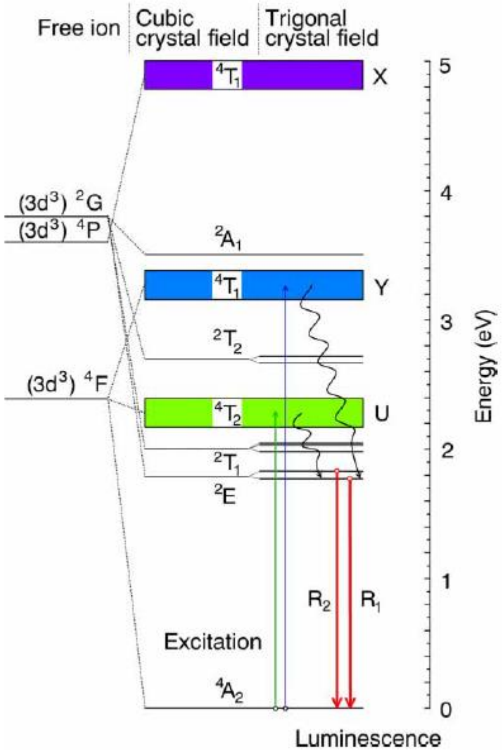
$$P_{Brown}-P_{Decker}/P \approx 3\%$$

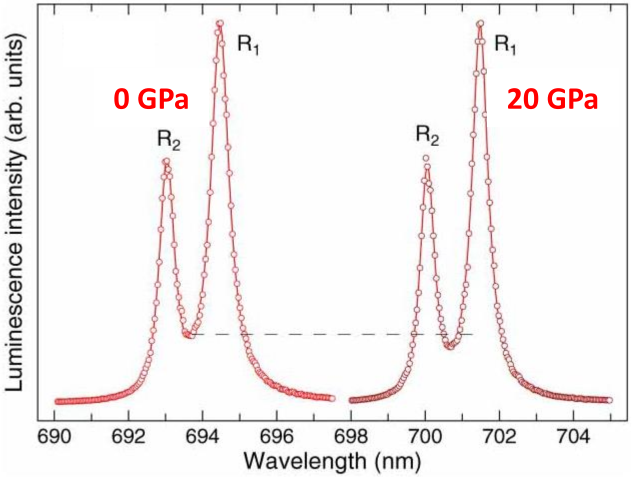
Probably more accurate than Decker!

The ruby fluorescence gauge

Ruby: $Al_2O_3:Cr^{3+}$ (« corundum »)

Cr^{3+}: ~ 0.1-1 at%





$$\boxed{\Delta\lambda/\Delta P = +0.365 \text{ nm/GPa}}$$

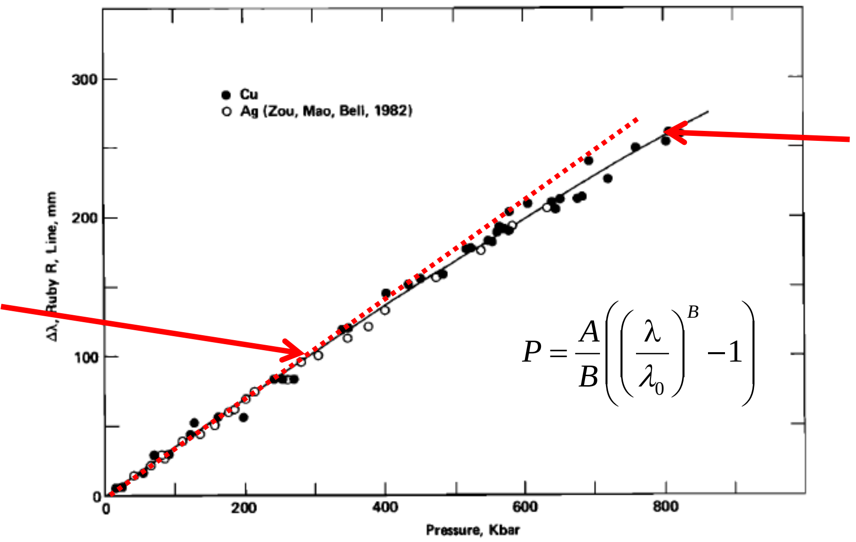
Piermarini et al. JAP 1975

K. Syassen, High Pres. Res. 28, 75 (2008)

The ruby 1986 calibration
(« Mao quasi-hydrostatic scale »)

Initial slope forced to be coherent with Decker scale

Mao, Xu, Bell, JGR 91, 4673 (1986)



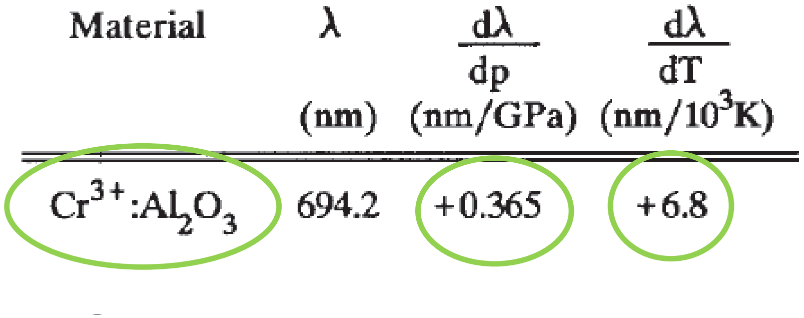
Curvature coherent with shock EOS data for V,Cu, Ag

- « The Decker scale is the mother of the ruby scale »

$$P = \frac{A}{B}\left(\left(\frac{\lambda}{\lambda_0}\right)^B - 1\right)$$

A = 1904 GPa
B = 7.665

Other fluorescence gauges



Material	λ (nm)	$\frac{d\lambda}{dp}$ (nm/GPa)	$\frac{d\lambda}{dT}$ (nm/10^3K)
$Cr^{3+}:Al_2O_3$	694.2	+0.365	+6.8

Ruby:

Moderate $d\lambda/dp$

Large $d\lambda/dT$

Pressure transmitting media & hydrostatic conditions

Why is this important?

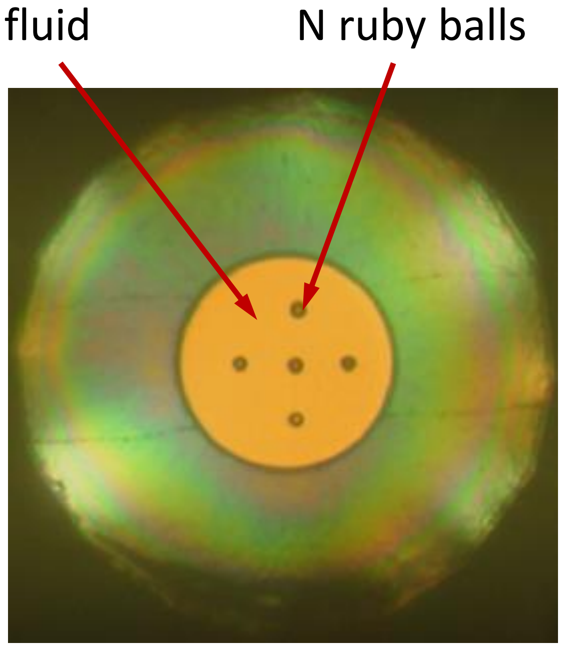
- « Pressure » in physics means almost exclusively <u>hydrostatic</u> pressure: $\sigma = -\delta_{ij}\, p$

  ex: $p = -(\partial E/\partial V)_T$ $B = -(\partial p/\partial \ln V)_T$

- Hydrostatic P is reproducible and easy to characterize other P-conditions not/only with great difficulties

Characterizing hydrostaticity

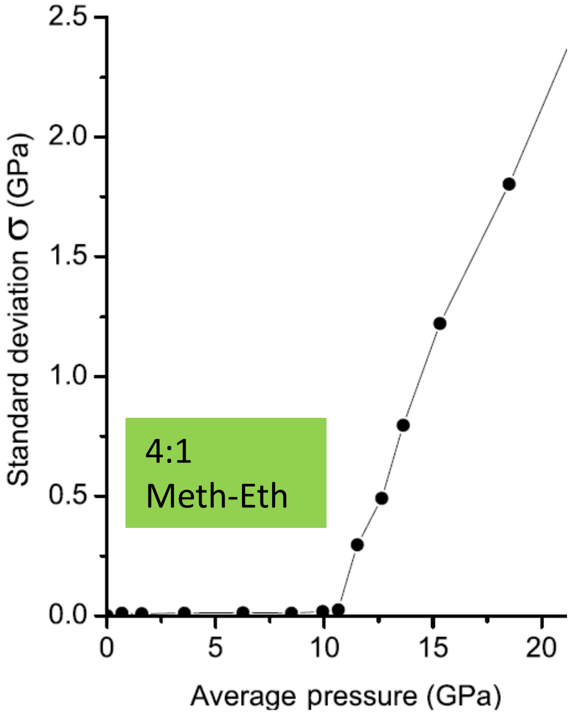
Example 4:1 methanol–ethanol (Piermarini 1973)



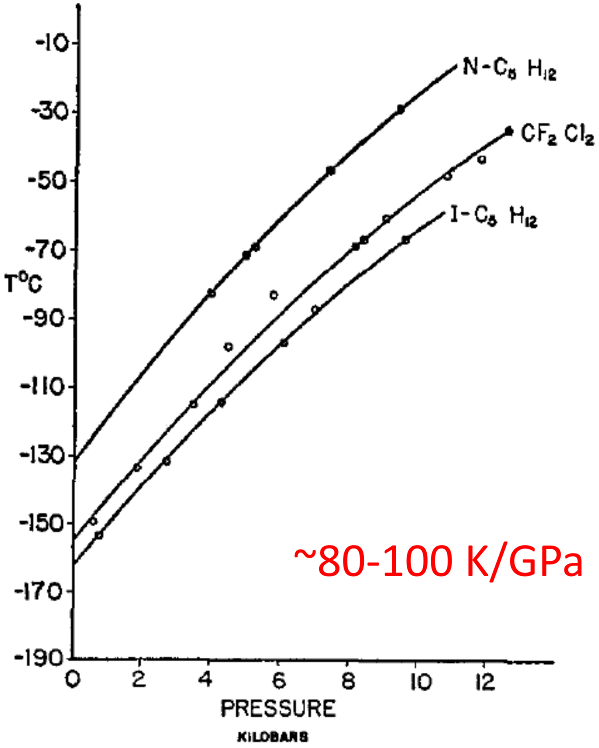
© K. Takemura

S.K., J.C. Chervin, P. Munsch, G. Le Marchand,
J. Phys. D: Appl. Phys. 42, 07413 (2009)

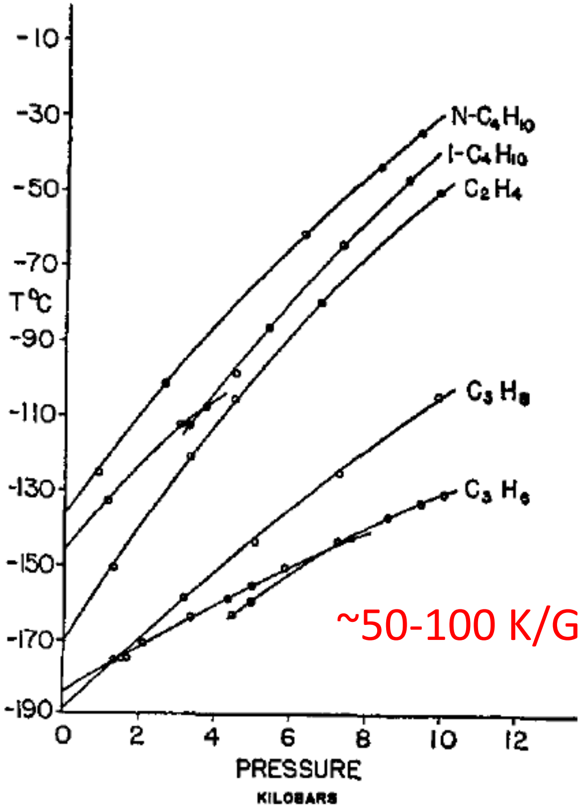
$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - \bar{P})^2}$$



Melting curves: pure substances



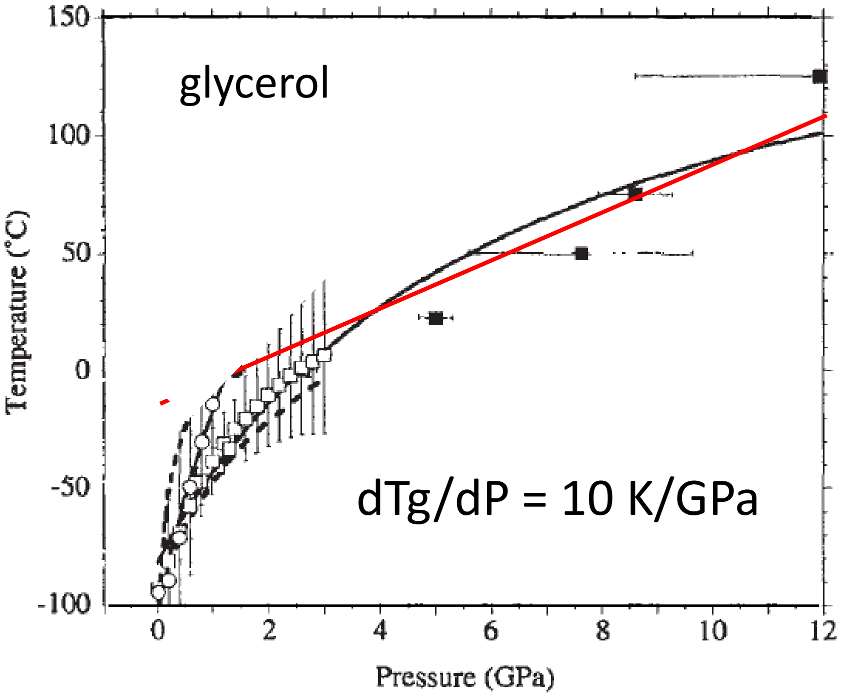
~80-100 K/GPa



~50-100 K/GPa

Reeves et al. JCP 1964

Glass forming liquids: P-dependence of T_g



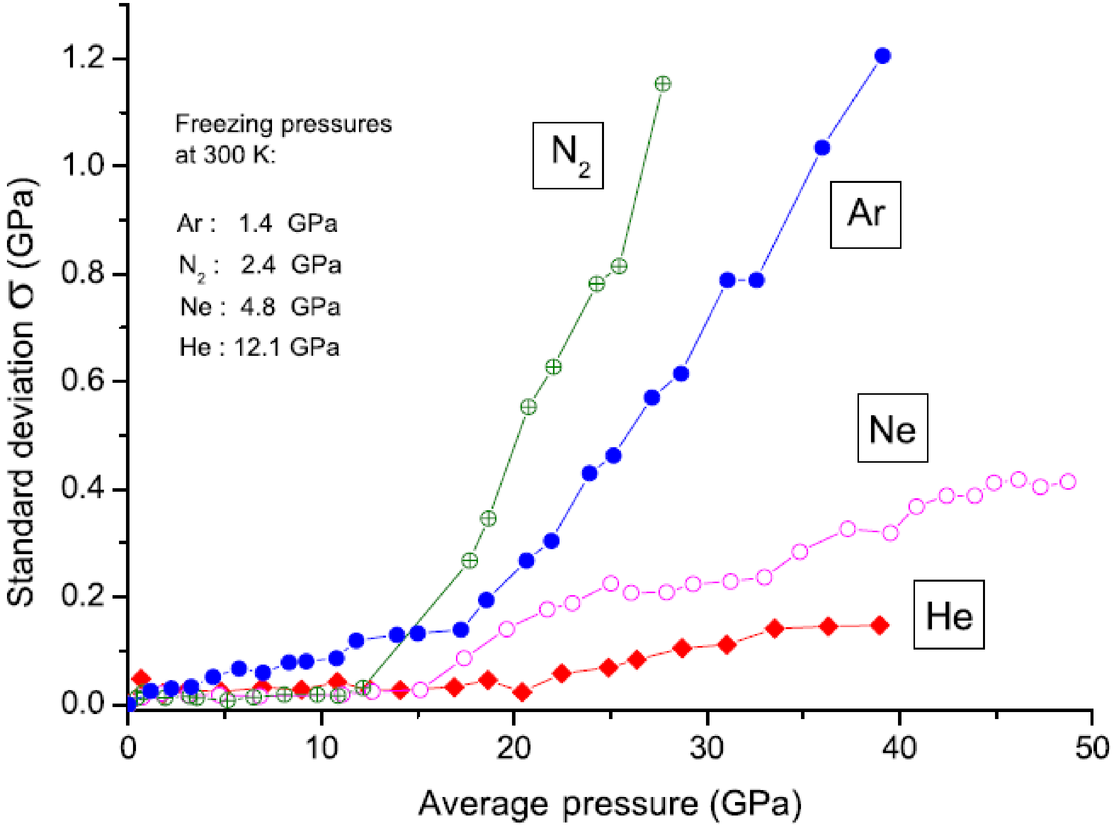
Cook et al. , JCP 1994

4:1 Meth-Eth: ≈ 11 K/GPa

DAPHNE: ≈ 25 K/GPa

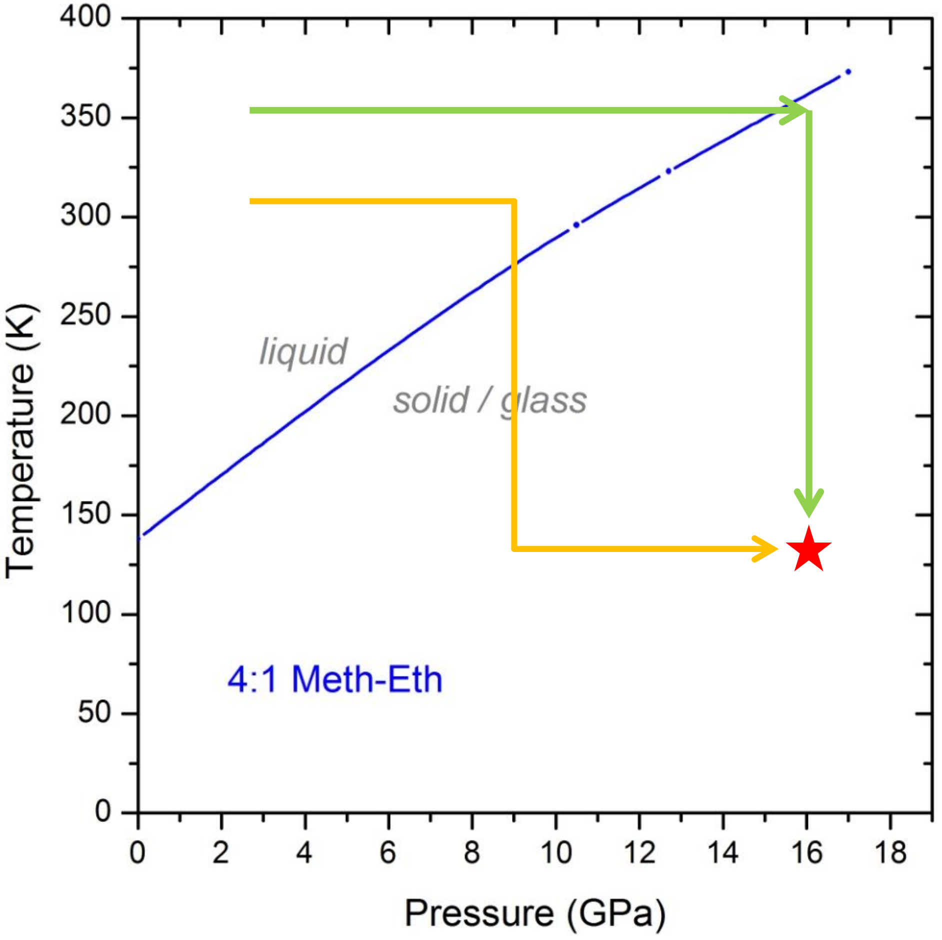
Much smaller than melting lines!

Compressed gases as PTM



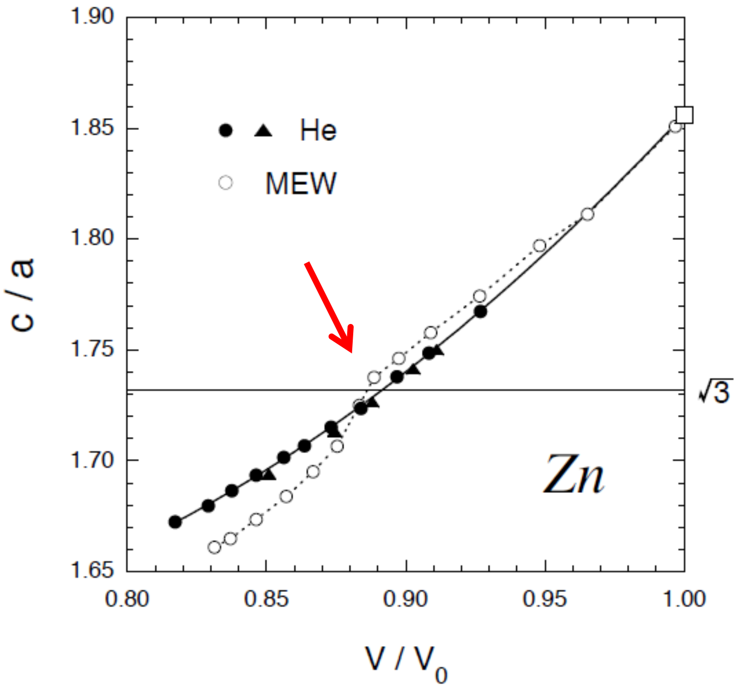
S.K., J.C. Chervin, P. Munsch, G. Le Marchand,
J. Phys. D: Appl. Phys. 42, 07413 (2009)

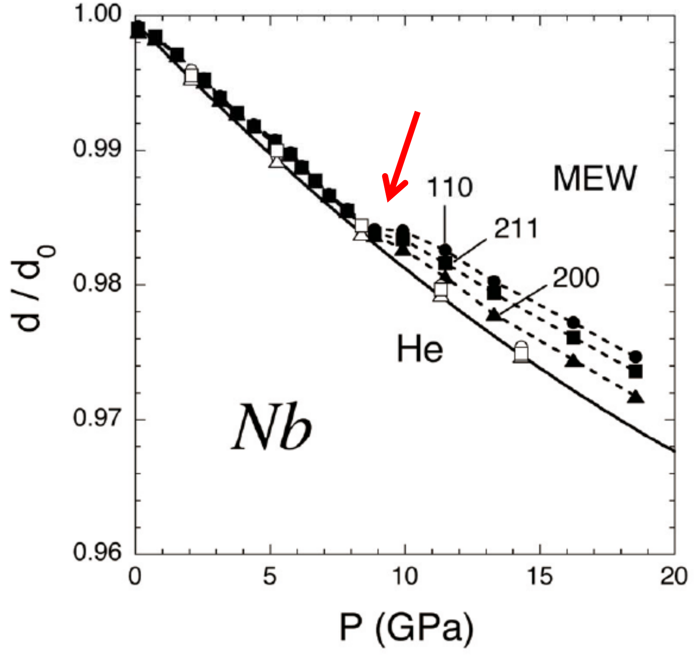
ME: Optimizing compression procedure



4:1 Methanol-Ethanol, see:, S.K. et al., High Press. Res. 29, 649 (2009)

Pitfalls I: Freezing of the PTM



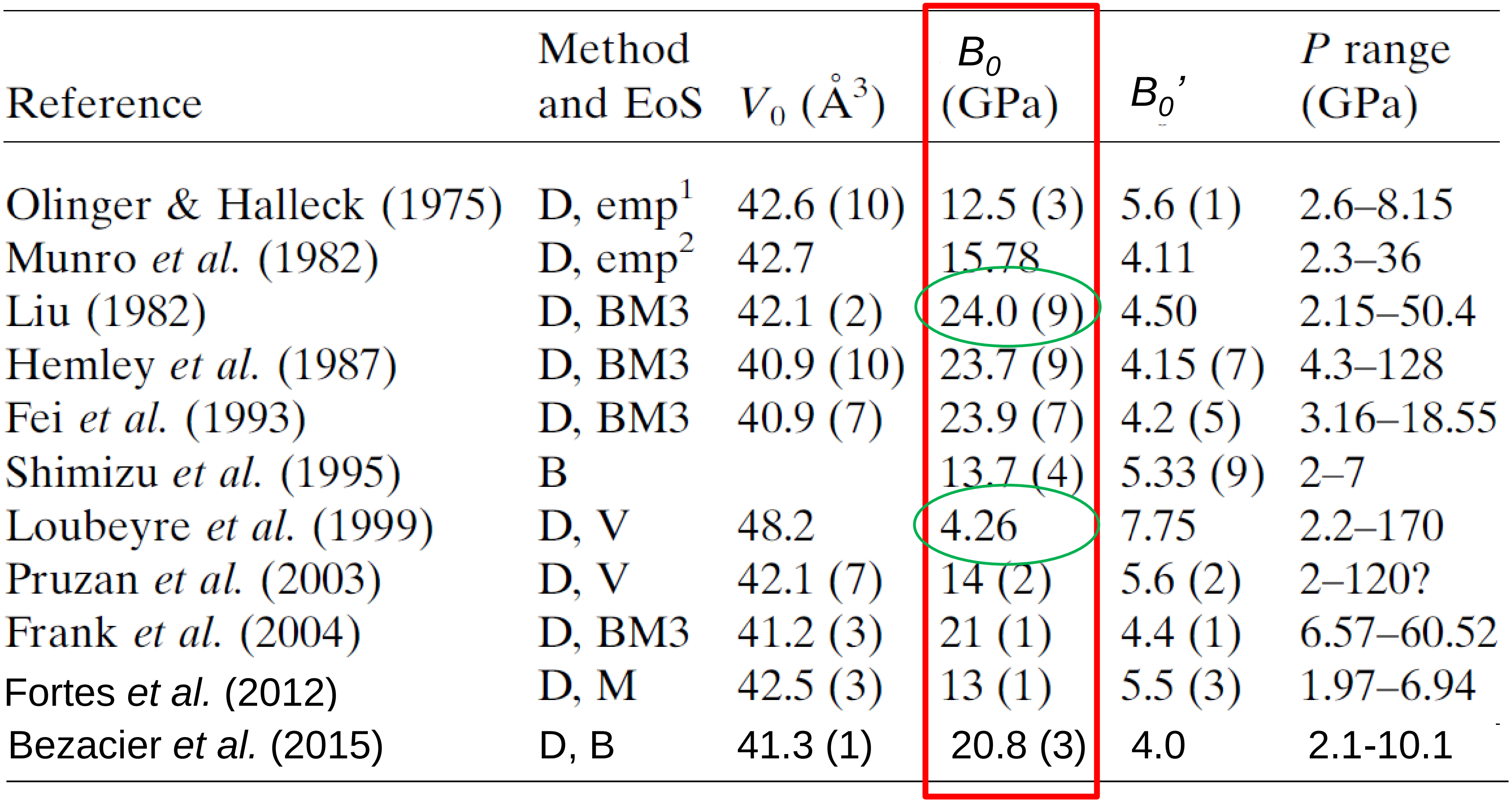


K. Takemura, PRL 1995
K. Takemura, Proc. AIRAPT Conf. 2005

K. Takemura, JPSJ, 2007

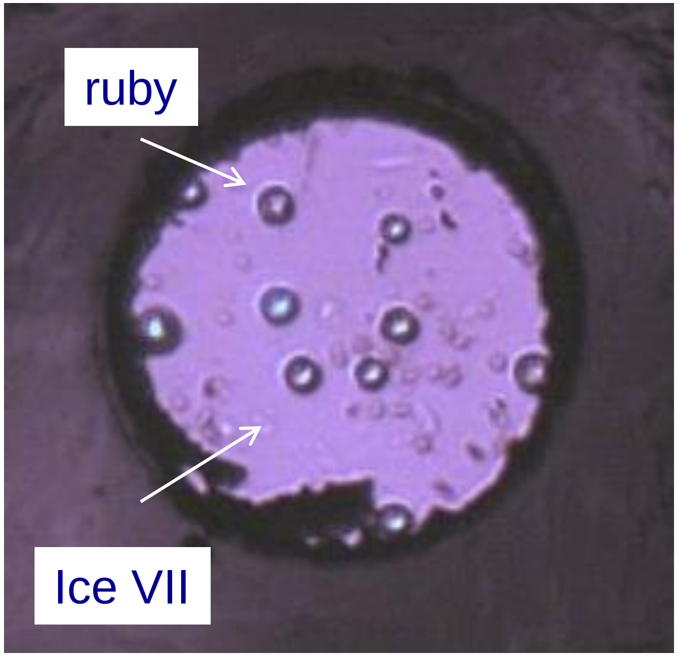
Pitfalls II: No PTM

Example: EOS of ice VII

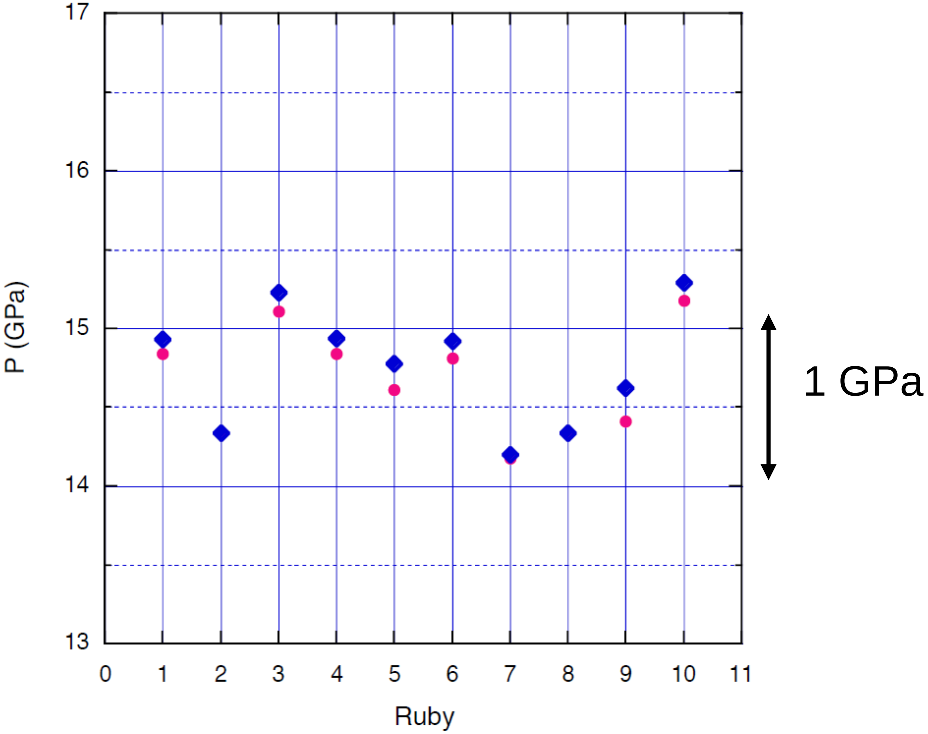


Reference	Method and EoS	V_0 (Å^3)	B_0 (GPa)	B_0'	P range (GPa)
Olinger & Halleck (1975)	D, emp[1]	42.6 (10)	12.5 (3)	5.6 (1)	2.6–8.15
Munro *et al.* (1982)	D, emp[2]	42.7	15.78	4.11	2.3–36
Liu (1982)	D, BM3	42.1 (2)	24.0 (9)	4.50	2.15–50.4
Hemley *et al.* (1987)	D, BM3	40.9 (10)	23.7 (9)	4.15 (7)	4.3–128
Fei *et al.* (1993)	D, BM3	40.9 (7)	23.9 (7)	4.2 (5)	3.16–18.55
Shimizu *et al.* (1995)	B		13.7 (4)	5.33 (9)	2–7
Loubeyre *et al.* (1999)	D, V	48.2	4.26	7.75	2.2–170
Pruzan *et al.* (2003)	D, V	42.1 (7)	14 (2)	5.6 (2)	2–120?
Frank *et al.* (2004)	D, BM3	41.2 (3)	21 (1)	4.4 (1)	6.57–60.52
Fortes *et al.* (2012)	D, M	42.5 (3)	13 (1)	5.5 (3)	1.97–6.94
Bezacier *et al.* (2015)	D, B	41.3 (1)	20.8 (3)	4.0	2.1-10.1

(adapted from Fortes 2012)



Takemura & Klotz, unpublished



Pitfalls III: Penetration

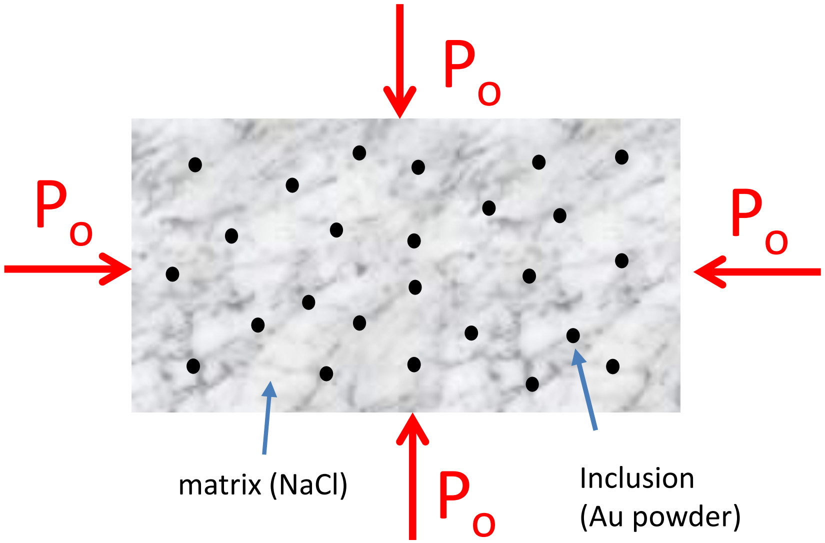
Many many other examples: ex: zeolites, glasses

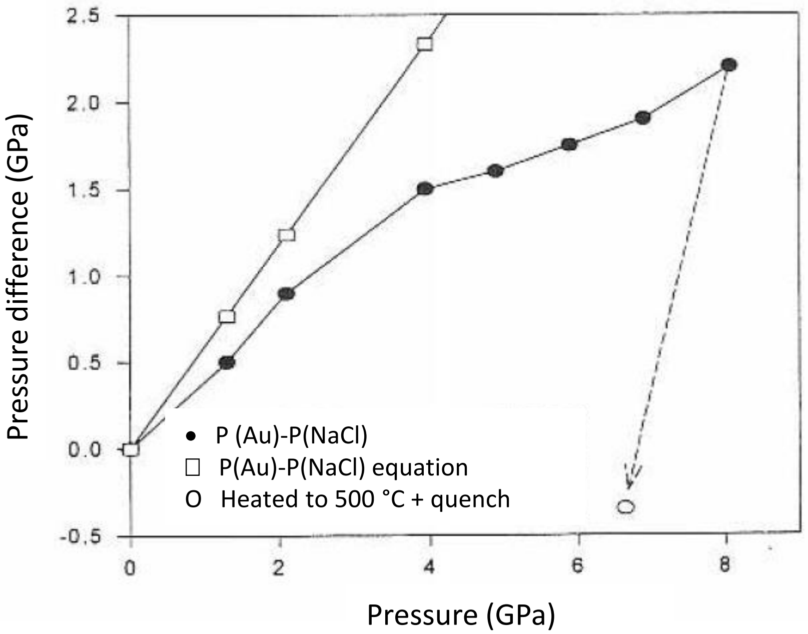
Message: check crystal structure first!

Pitfalls IV: The Lamé effect

→ solid PTMs

Ex: Au powder in NaCl





Wang & Weidner, Geophy. Monogr., 1998

$$P_i = P_o\left(1 + \frac{4\mu_m}{3B_m}\right) / \left(1 + \frac{4\mu_m}{3B_i}\right)$$

Thank you !
Questions ?