Optimal positioning of a single local magnetic sensor for integrated dipole measurements

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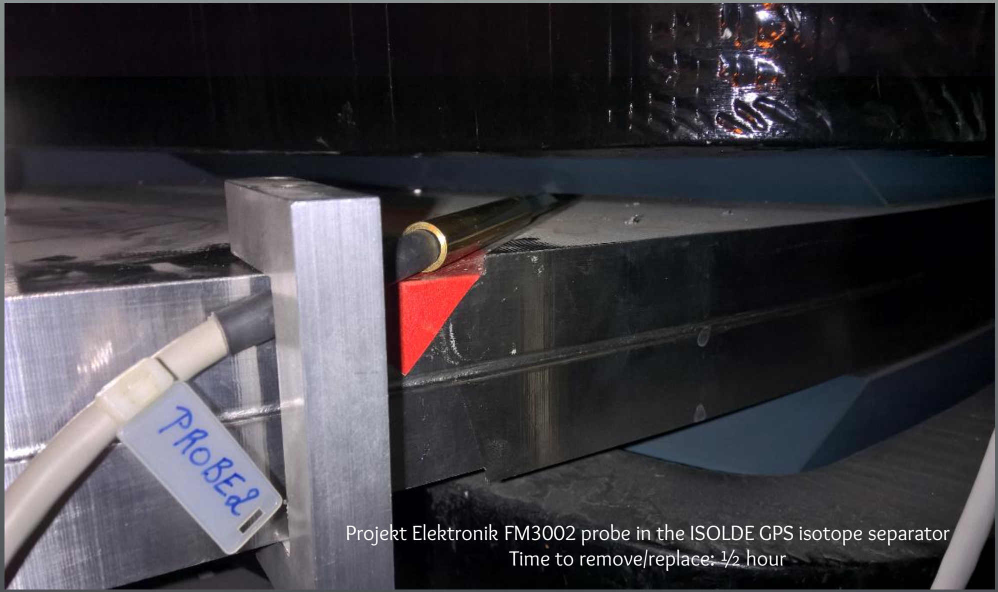
Contents

- (a beam physicist's) **problem**: use a single sensor to infer a dipole's field integral
  (typical application: real-time control of single experimental spectrometers or transfer line magnets)
- (his typical) **solution**: grab the first probe you find and stick it inside as deep as it will go



Projekt Elektronik FM3002 probe in the ISOLDE GPS isotope separator
Time to remove/replace: ½ hour

ISOLDE HRS isotope separator
Dose at contact in ½ hour: 0.5 mSv

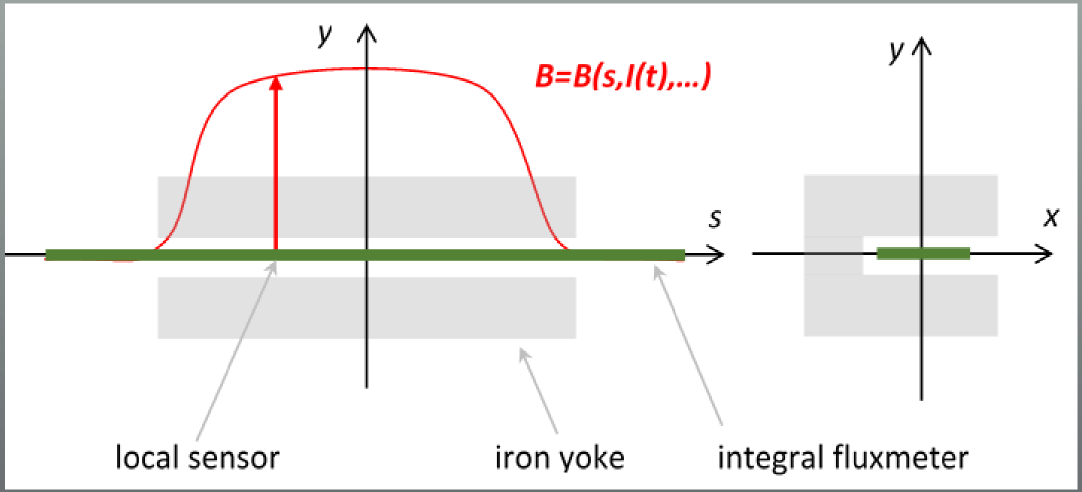
- The usual outcome:

- *Is there a better way ?*

detailed answer here: M. Buzio, G. Golluccio, C. Grech, S. Russenschuck, N. Sammut, *"Optimal positioning of a single local magnetic sensor for integrated dipole measurements"*, submitted to Physical Review Accelerators and Beams

Part I
Magnetic model

Magnetic length



- Initial assumptions:
  - no dynamic or history-dependent effects
  - consider only sensor positions along the nominal beam path s
- Define a *generalized* magnetic length: $$\ell_m(s,I) = \frac{\int_{-\infty}^{\infty} B(s,I)ds}{B(s,I)}$$
- Commonly used definition: $$\ell_m = \ell_m(0) = const.$$

Goal: find the optimal position s^* where the variation of ℓ_m w.r.t. I (or other variables) is minimal

Analytical field profile model

- **Simplest model:** assume superposition of two profile components:
  - $\lambda(s)$, scaling linearly with the current (associated with the coil and the contribution of iron at low field)
  - $\sigma(s)$, scaling non-linearly (associated with saturating iron regions)

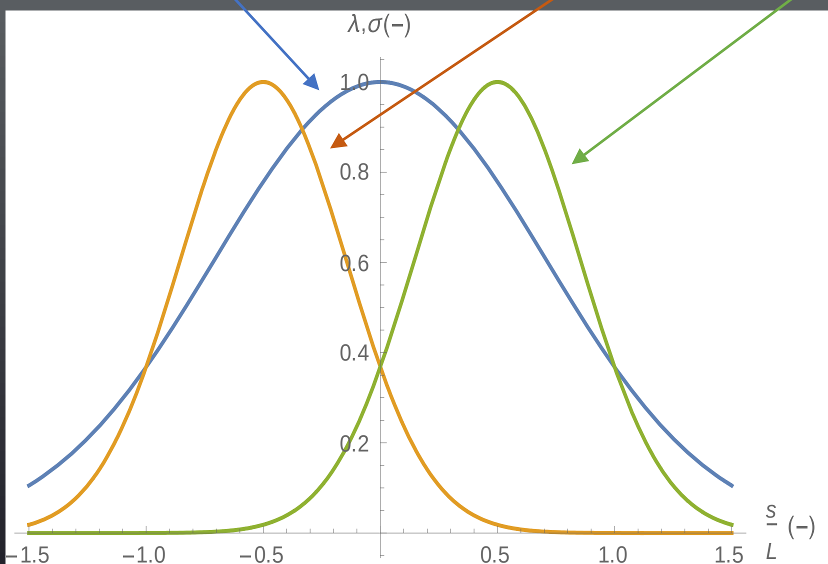
$$B(s,I) = B_0 \frac{I}{I_0}\left(\lambda(s) + \varsigma\left(\frac{I}{I_0}\right)\sigma(s)\right)$$

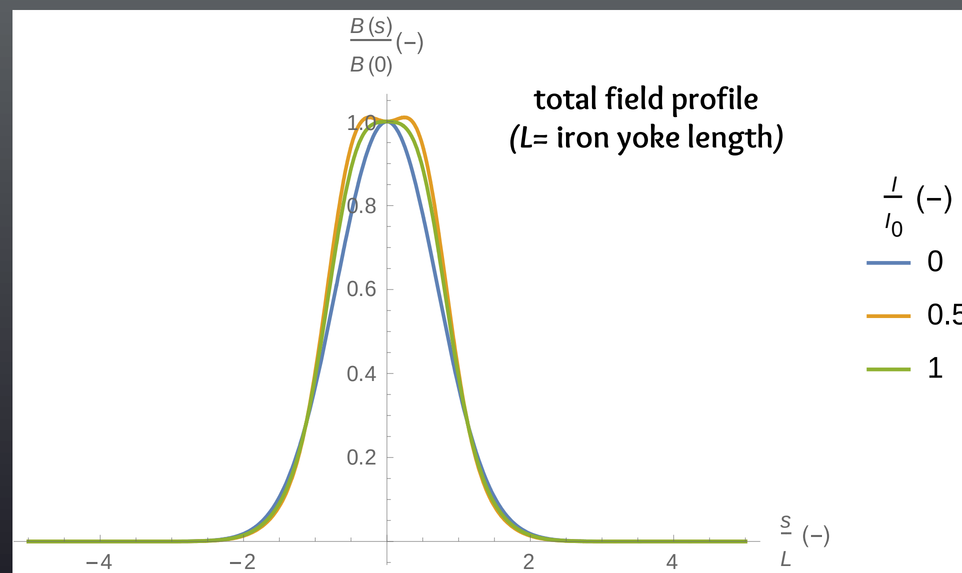
reference (normalization) values

saturation characteristic e.g. $\varsigma\left(\frac{I}{I_0}\right) = 1 - \alpha\left(\frac{I}{I_0}\right)^n$

- Reasonable choice: **bell-shaped curves** for the profile, e.g. $\varphi(s; s_0, \eta) = \mathrm{e}^{-\frac{(s-s_0)^2}{\eta^2 L^2}}$

$\lambda(s) = \varphi(s; 0, \eta_L)$ $\quad$ $\sigma(s) = \varphi\left(s; -\frac{L}{2}, \eta_S\right) + \varphi\left(s; \frac{L}{2}, \eta_S\right)$





Optimal magnetic length

- **Magnetic length** vs. position s and current I:

$$\ell_m(s,I)=\frac{\int_{-\infty}^{\infty}B(s,I)ds}{B(s,I)}=\frac{\int_{-\infty}^{\infty}\lambda(s)ds+\varsigma\left(\frac{I}{I_0}\right)\int_{-\infty}^{\infty}\sigma(s)ds}{\lambda(s)+\varsigma\left(\frac{I}{I_0}\right)\sigma(s)}$$

- Stationarity of ℓ_m:

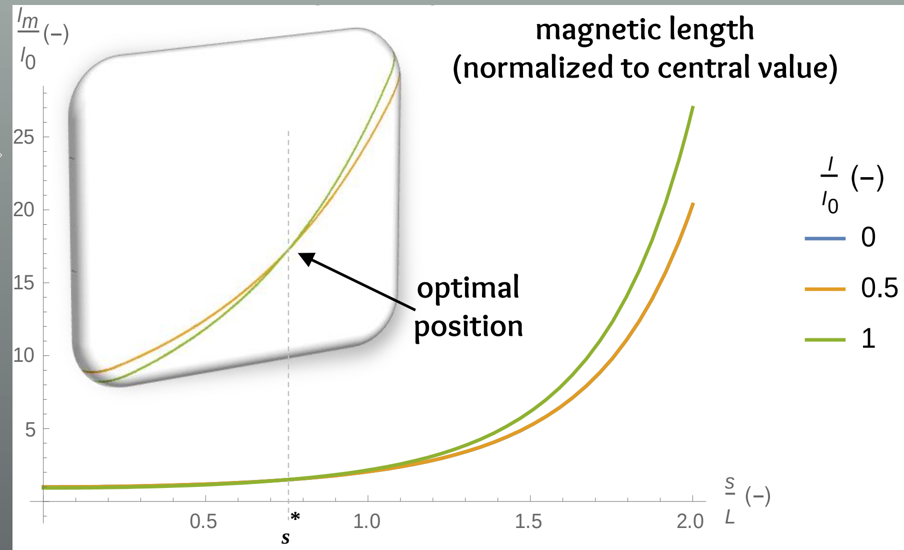
$$\frac{\partial\ell_m}{\partial I}=\frac{1}{I_0}\frac{\lambda(s)\int_{-\infty}^{\infty}\sigma(s)ds-\sigma(s)\int_{-\infty}^{\infty}\lambda(s)ds}{\left(\lambda(s)+\varsigma\left(\frac{I}{I_0}\right)\sigma(s)\right)^2}\varsigma'\left(\frac{I}{I_0}\right)=0$$

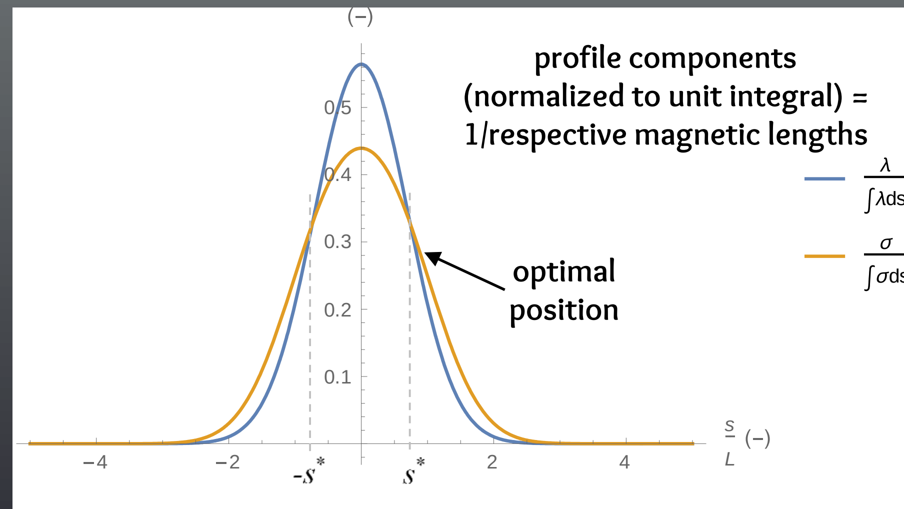
- **Optimal sensor position:**

$$\ell_m^*=\ell_m(s^*)=\frac{\int_{-\infty}^{\infty}\lambda(s)ds}{\lambda(s^*)}=\frac{\int_{-\infty}^{\infty}\sigma(s)ds}{\sigma(s^*)}$$

- **Analytical solution** with bell-shaped profiles and $\eta_L >> \eta_s$:

$$s^*=\frac{L}{2}\frac{1\pm\sqrt{1-\left(1-\frac{\eta_S^2}{\eta_L^2}\right)\left(1+4\eta_S^2\ln 2\frac{\eta_S}{\eta_L}\right)}}{1-\frac{\eta_S^2}{\eta_L^2}},\qquad \lim_{\eta_S\to 0}s^*=\pm\frac{1}{2}$$





for arbitrary $\lambda(s)$, $\sigma(s)$ $\exists s^*:\ell_m(s^*)$ is a constant
vanishingly thin saturating region → s^* coincides with edge of yoke

Relaxing the assumptions

Impact of:

- **remanent field:**

remanent field profile

$$B(s,I) = B_0\left(\rho(s) + \frac{I}{I_0}\left(\lambda(s) + \varsigma\left(\frac{I}{I_0}\right)\sigma(s)\right)\right)$$

additional terms in *I* appear, unless $\rho(s)$ has same *shape* as either $\lambda(s)$ or $\sigma(s)$

$$\ell_m(s^*,I) = \frac{\int_{-\infty}^{\infty}\lambda(s)ds}{\lambda(s^*)} + \frac{1}{2}\frac{\int_{-\infty}^{\infty}\lambda(s)ds\int_{-\infty}^{\infty}\sigma(s)ds}{\left(\int_{-\infty}^{\infty}\lambda(s)ds + \int_{-\infty}^{\infty}\sigma(s)ds\right)^2}\left(\frac{\int_{-\infty}^{\infty}\lambda(s)ds}{\lambda(s^*)} - \frac{\int_{-\infty}^{\infty}\rho(s)ds}{\rho(s^*)}\right)\varsigma''\left(\frac{I}{I_0}\right)\frac{\rho(s^*)}{\lambda(s^*)}\frac{I}{I_0} + \cdots$$

- **eddy currents:**

$$\begin{cases} B(s,I,t) = B_0\left(\frac{I_e(t)}{I_0}\varepsilon(s) + \frac{I}{I_0}\left(\lambda(s) + \varsigma\left(\frac{I}{I_0}\right)\sigma(s)\right)\right) \\ \tau_e\frac{dI_e}{dt} + I_e = -\tau_{em}\frac{dI}{dt} \end{cases} \Rightarrow B(s,I) = B_0\left(-\tau_{em}\frac{\dot{I}}{I_0}\varepsilon(s) + \frac{I}{I_0}\left(\lambda(s) + \varsigma\left(\frac{I}{I_0}\right)\sigma(s)\right)\right)$$

same situation as above

- **multiple saturating regions:**

$$B(s,I) = B_0\frac{I}{I_0}\left(\lambda(s) + \sum_{k=1}^{N}\varsigma_k\left(\frac{I}{I_0}\right)\sigma_k(s)\right)$$

e.g. N=2:

additional terms in *I* appear, unless all $\sigma_k(s)$ have the same *shape*

$$\ell_m(s^*,I) = \frac{\int_{-\infty}^{\infty}\sigma_1(s)ds + \int_{-\infty}^{\infty}\sigma_2(s)ds}{\sigma_1(s^*) + \sigma_2(s^*)}\left(1 + \gamma\epsilon\frac{I}{I_0} + \cdots\right)$$

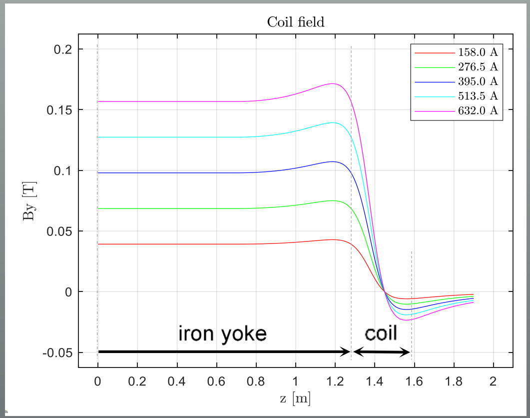
$$\gamma = \frac{\sigma_1(s^*)\sigma_2(s^*)}{\sigma_1(s^*) + \sigma_2(s^*)}\frac{\frac{\int_{-\infty}^{\infty}\sigma_2(s)ds}{\sigma_2(s^*)} - \frac{\int_{-\infty}^{\infty}\sigma_1(s)ds}{\sigma_1(s^*)}}{\lambda(s^*) + \left(\sigma_1(s^*) + \sigma_2(s^*)\right)\varsigma_1\left(\frac{I}{I_0}\right)}$$

dynamic/hysteresis effects *or* multi-component model ⇒ the optimal magnetic length cannot be a constant

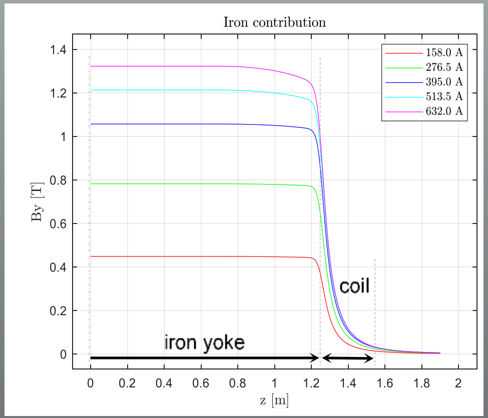
Part II
FE simulations

ISR dipole field profile

ROXIE FE simulation



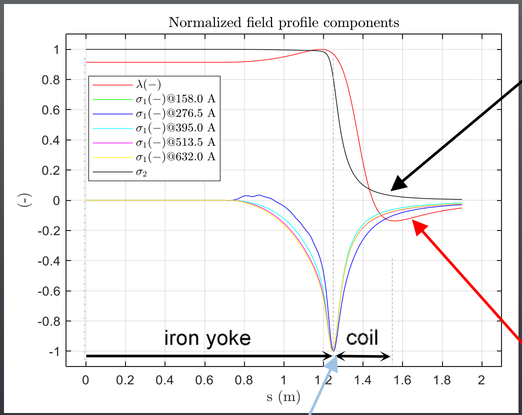
linear coil contribution $B_c(s)$



reduced field $B_r(s)$

Analytical model with two nonlinear components:

$$B(s,I) = \frac{I}{I_0} B_c(s,I_0) + B_r(s,I) = B_0 \frac{I}{I_0}\left(\lambda(s) + \varsigma_1\left(\frac{I}{I_0}\right)\sigma_1(s) + \varsigma_2\left(\frac{I}{I_0}\right)\sigma_2(s)\right)$$



$$\sigma_2(s) = \frac{B_r(s,I_0)}{\max_s\left(B_r(s,I_0)\right)}$$

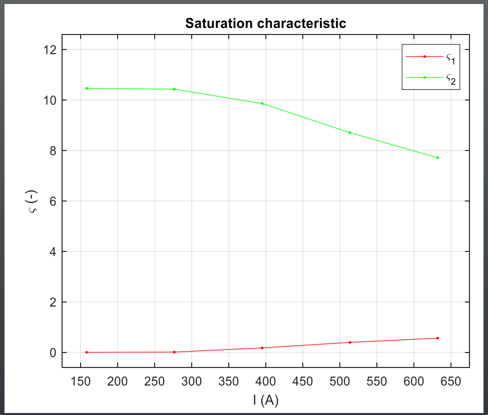
central region contribution

$I_0 = 158\ A$ (iron linear limit)

$$B_0 = \max_s\left(B_c(s,I_0)\right)$$

$$\lambda(s) = \frac{B_c(s,I_0)}{B_0}$$

linear coil contribution



this represents overall saturation:

$$\varsigma_2\left(\frac{I}{I_0}\right) = \frac{I_0}{I}\frac{\max_s\left(B_r(s,I)\right)}{B_0}$$

localized edge saturation:

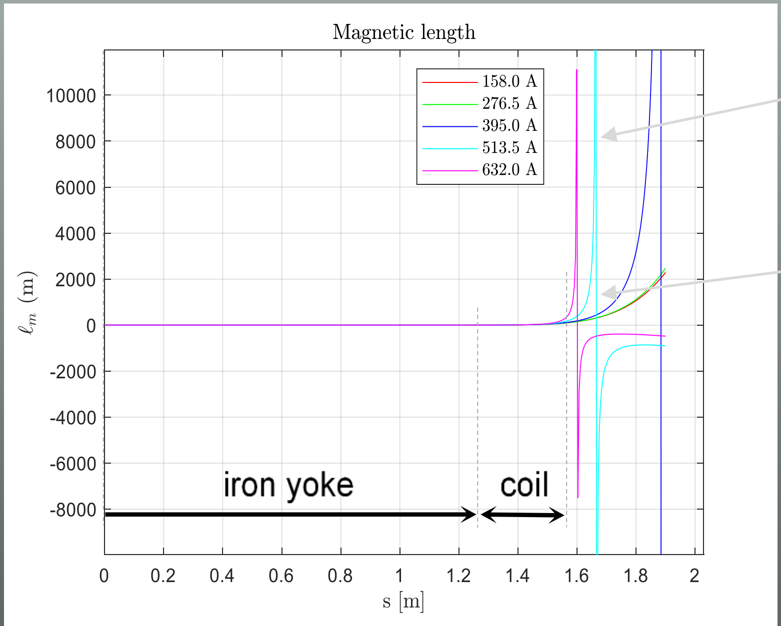
$$\varsigma_1\left(\frac{I}{I_0}\right) = \frac{I_0}{I}\frac{B_1}{B_0}$$

localized edge contribution

$$\sigma_1(s;I) = \frac{1}{B_1}\left(B_r(s,I) - \frac{\max_s\left(B_r(s,I)\right)}{\max_s\left(B_r(s,I_0)\right)} B_r(s,I_0)\right) \qquad B_1 = \max_s\left(B_r(s,I) - \frac{\max_s\left(B_r(s,I)\right)}{\max_s\left(B_r(s,I_0)\right)} B_r(s,I_0)\right)$$

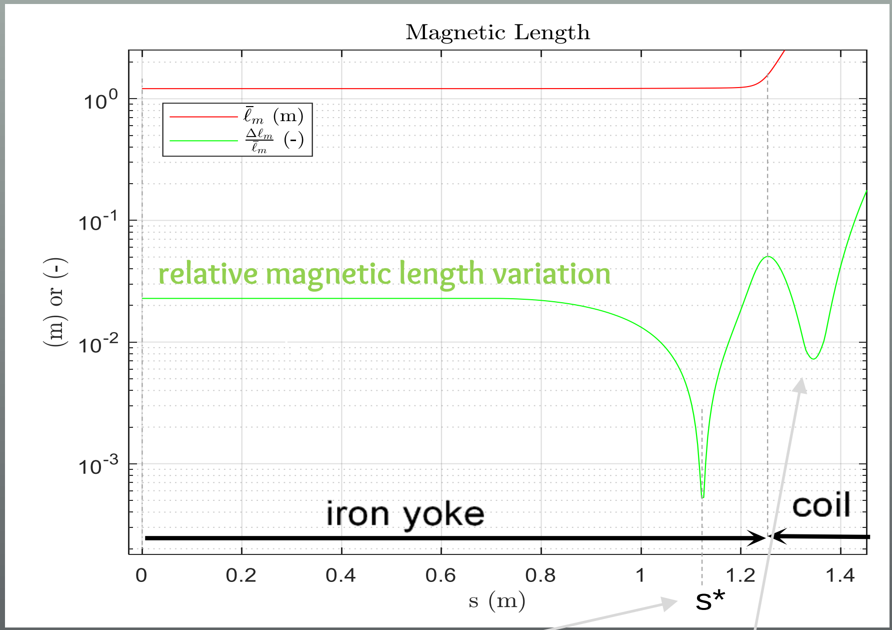
slight dependence of $\sigma_1(s)$ upon $I \Rightarrow$ oversimplified analytical model

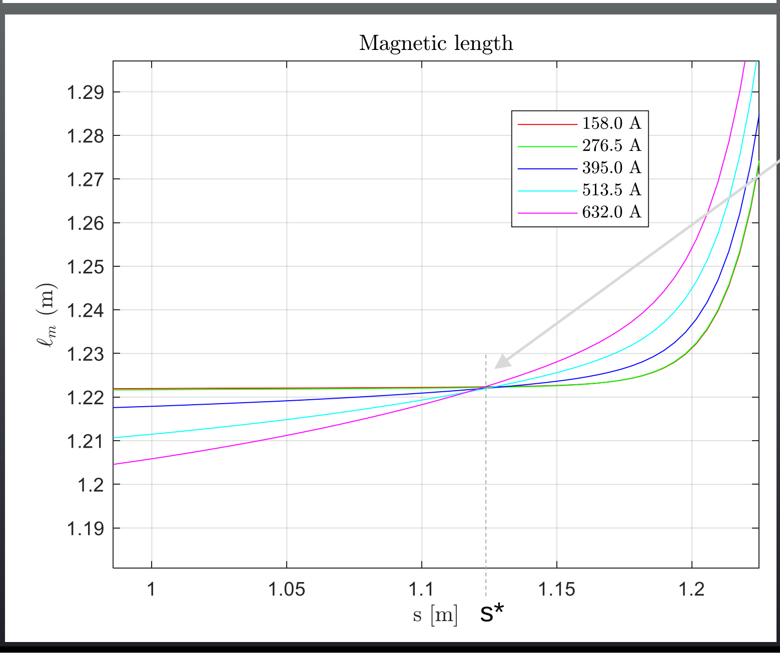
ISR dipole magnetic length



ℓ_m diverges as B → 0 in the fringe field region

the zero of B(s, *I*) shifts inwards with saturation



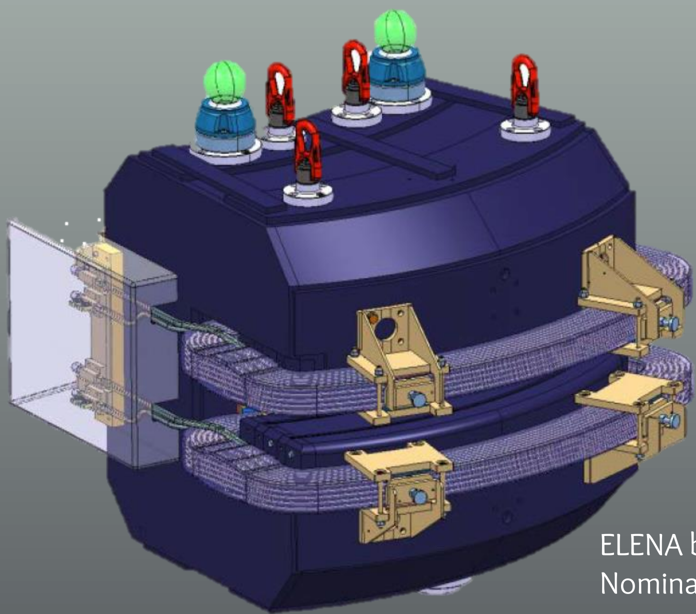


optimal sensor position ~120 mm from pole edge

secondary minimum in the fringe field region (useless!)

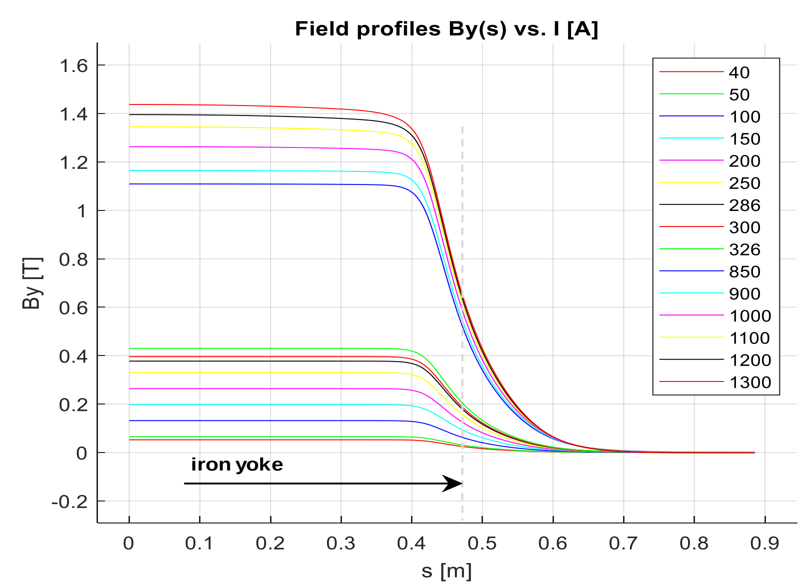
close qualitative match to analytical prediction theoretical 40× improvement at *s**

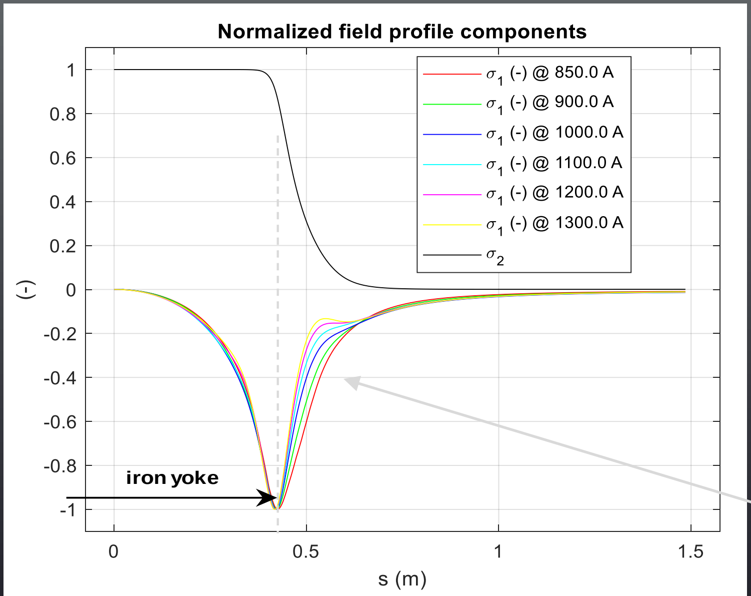
ELENA dipole field profile



ANSYS FE simulation
into extreme saturation region

ELENA bending dipole
Nominal: 0.45 T @ 326 A



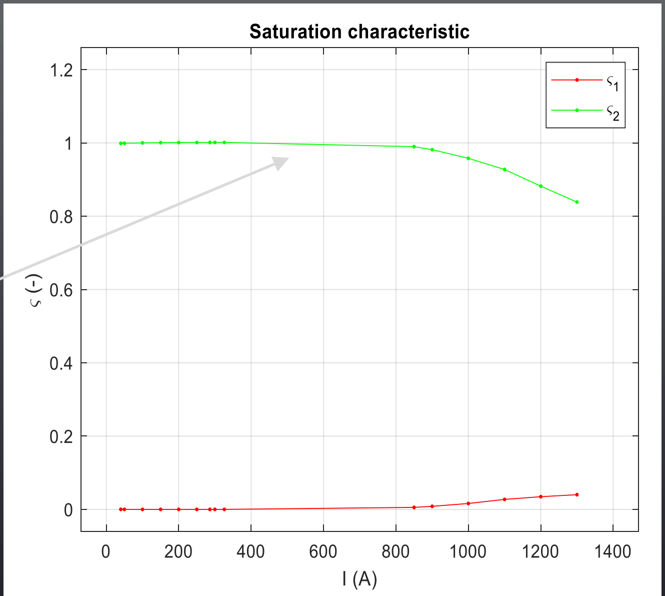


Analytical model with two nonlinear contributions:

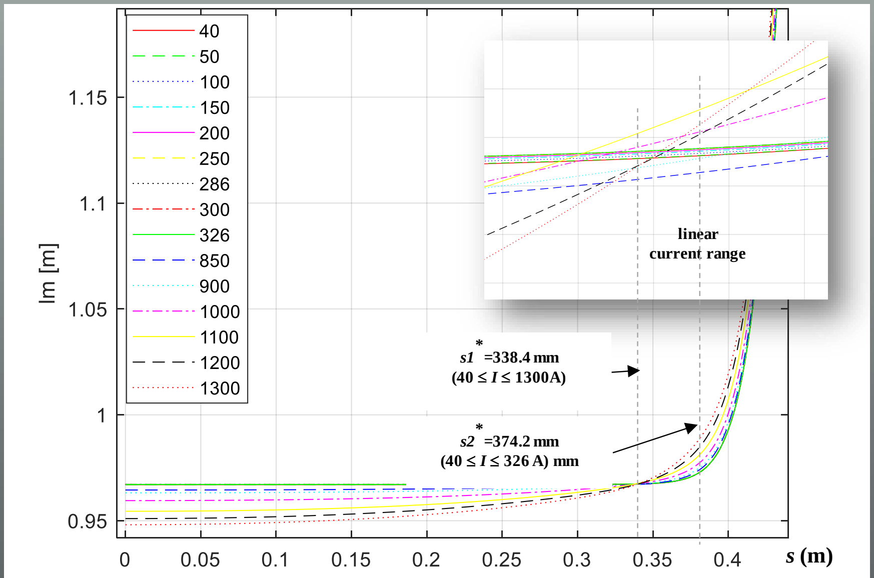
$$B(s,I) = B_0 \frac{I}{I_0}\left(\varsigma_1\left(\frac{I}{I_0}\right)\sigma_1(s) + \varsigma_2\left(\frac{I}{I_0}\right)\sigma_2(s)\right)$$

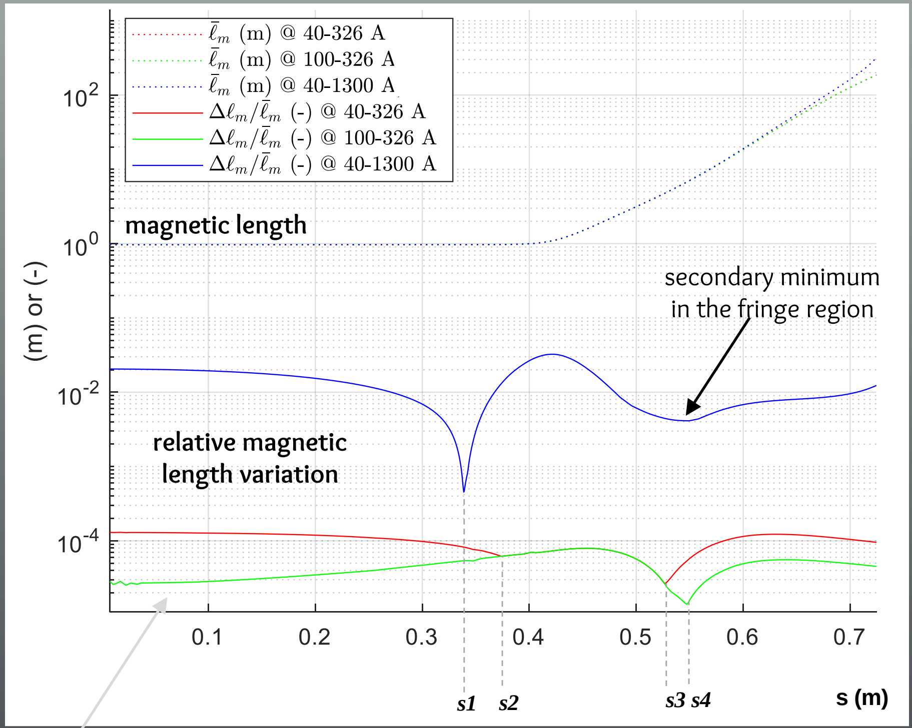
Bulk yoke contribution $\sigma_2(s)$
essentially linear well above nominal I

Edge contribution $\sigma_1(s)$ depends on I
(oversimplified model)

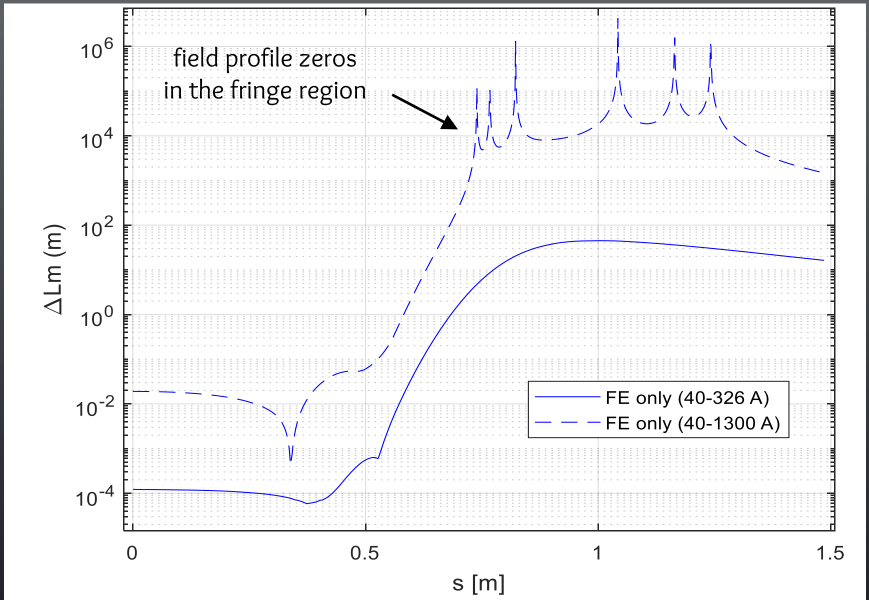


ELENA dipole magnetic length





sensor location hardly matters in the central region



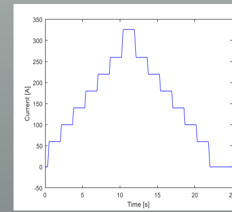
effect of saturation:
×100 magnetic length variation
sharper minima, shifted inwards

Part III
Test results

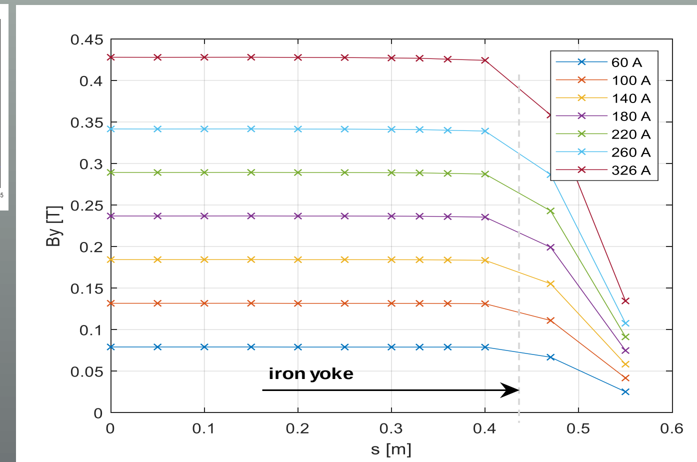
ELENA dipole: field profile

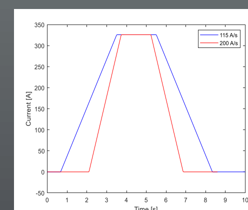
curved fluxmeter for the integral field (both dynamic and DC-equivalent at the end of each staircase plateau)

3 × Projekt Elektronik AS-NTM-2 Hall probes moved sequentially to 12 on-axis positions

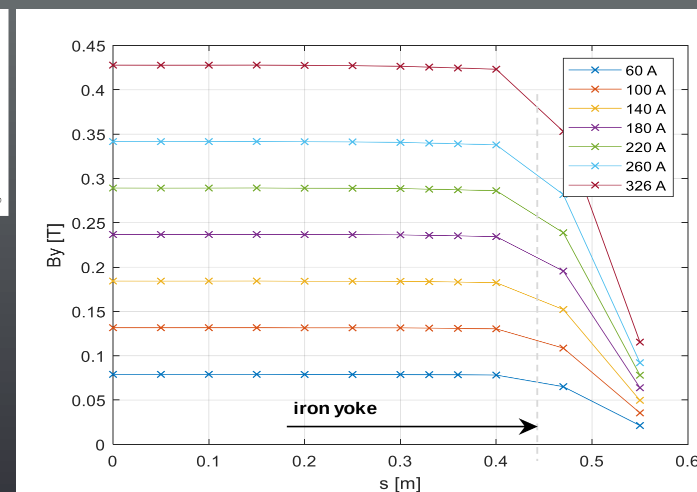


DC staircase (comparison to FE)

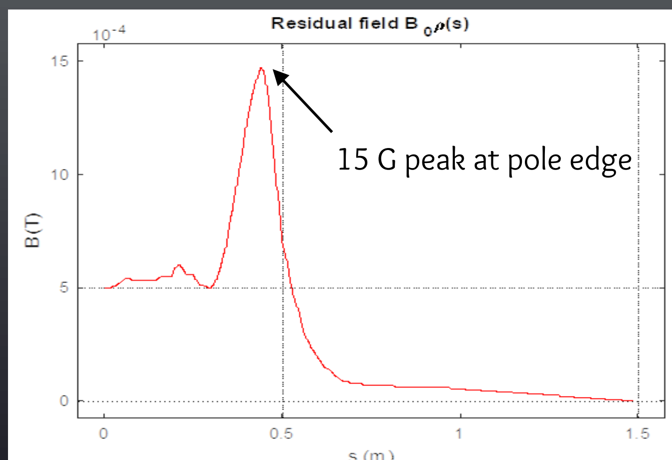




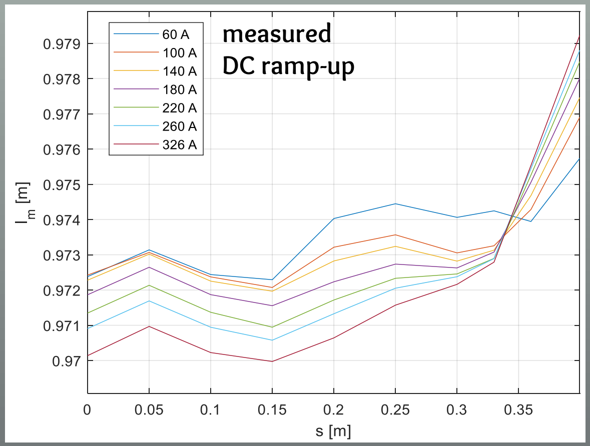
dynamic cycles @ 150 and 200 A/s (shown: 200) operational conditions

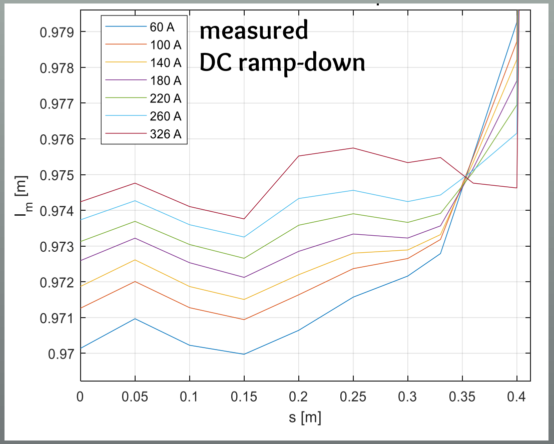


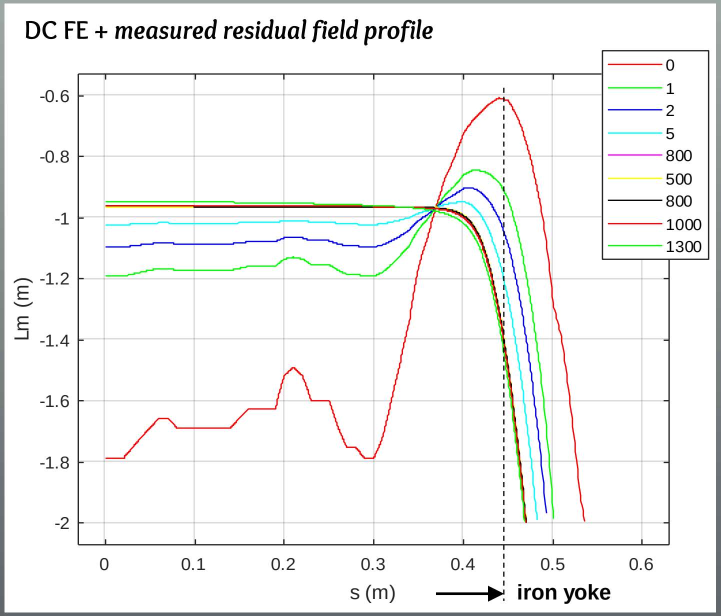
residual field profile measured Bartington MH-03 fluxgate used as integration constant (also added to FE computed profiles)

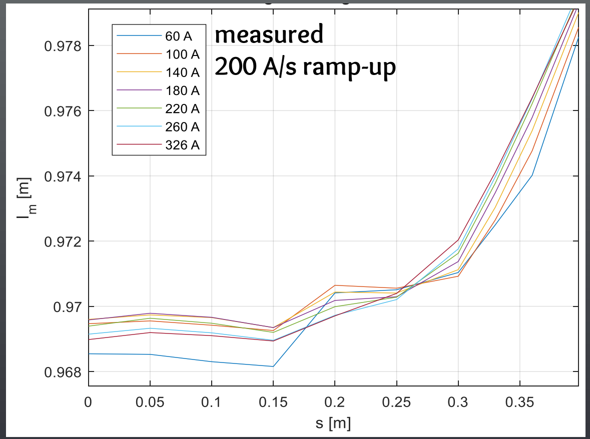


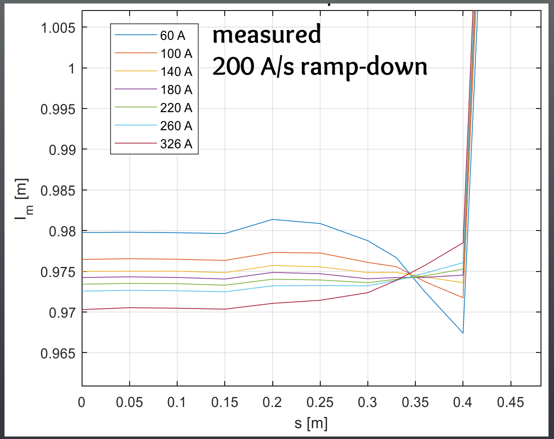
ELENA dipole: magnetic length

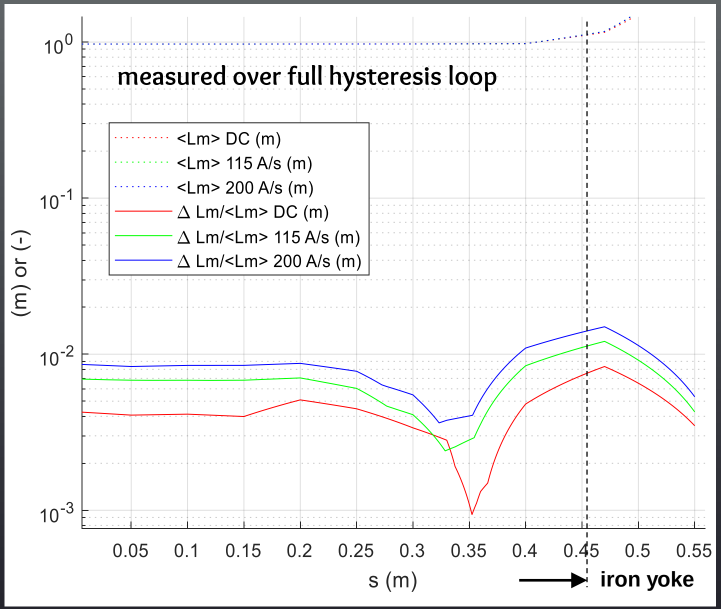








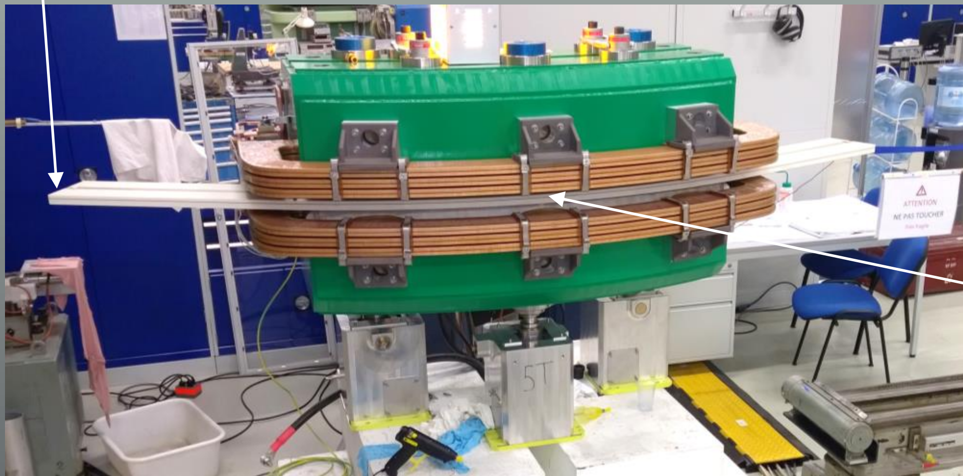




DC: measured s^* = 352 mm (FE: 369 mm)
200 A/s: measured s^* = 334 mm

ISOLDE dipole

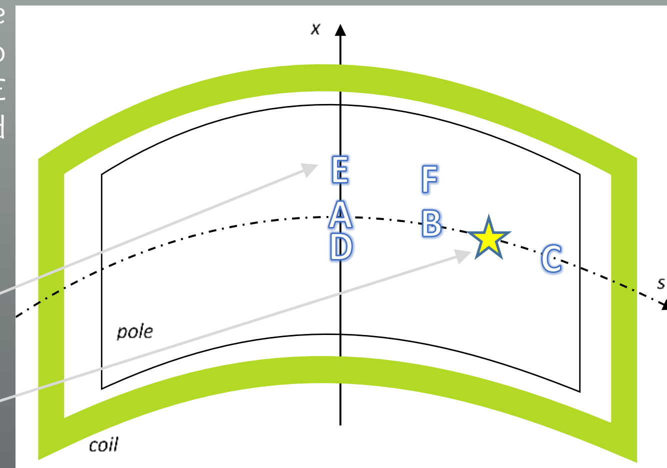
curved fluxmeter for the dynamic integral field (taken out/put back in for the remanent)



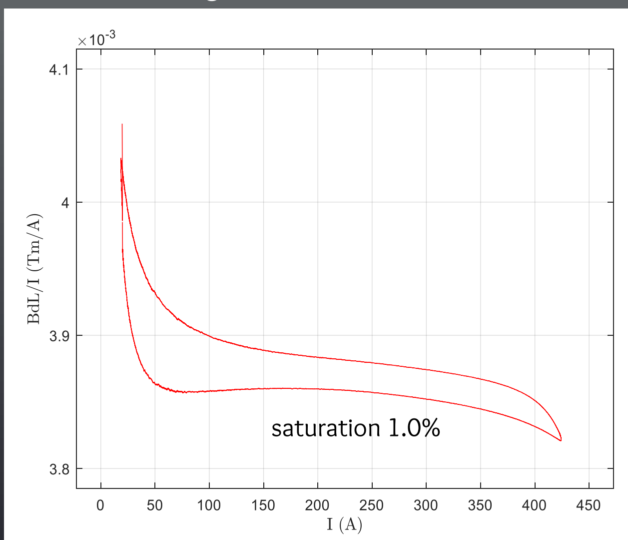
AREPOC Hall probe moved in sequence to six test positions A-E to measure the local field

best measured position (used for operation)

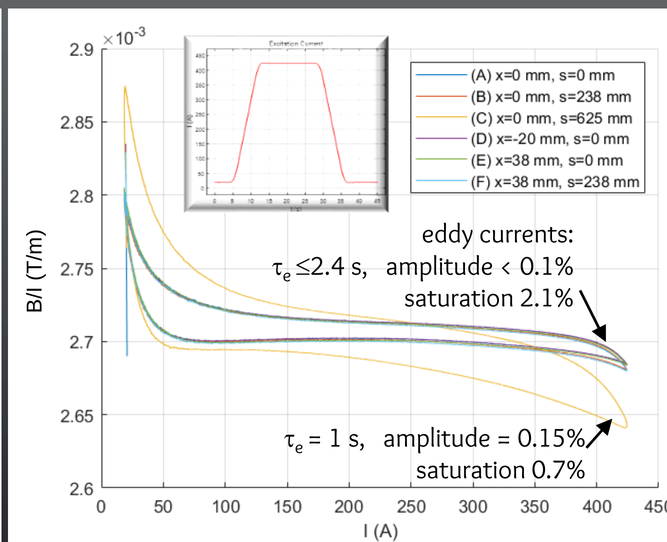
plausible optimal position



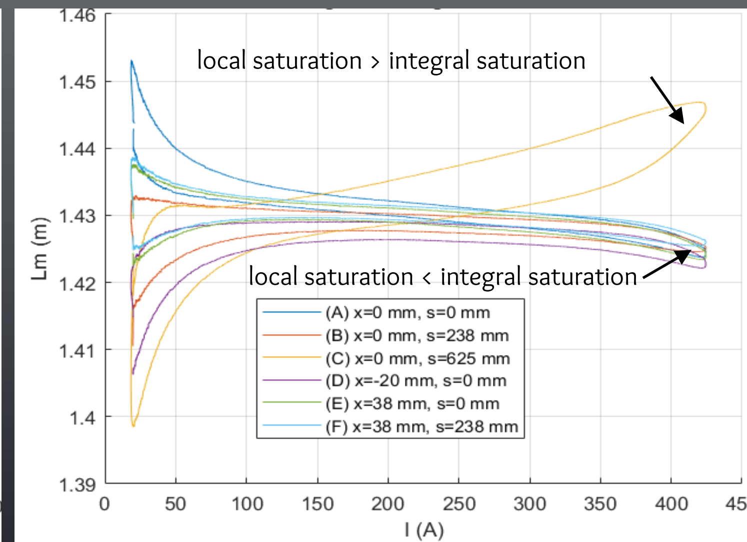
integral transfer function



local transfer function



magnetic length



results suggestive of strongly localized secondary eddy currents at the edge

ideally, local saturation = integral saturation

Conclusions

Conclusions

- Analytical and FE calculations prove that, under rather general assumptions, the **optimal position for a single Hall probe is towards the pole edge,** rather than at the center of the magnet

- **1-2 orders of magnitude improvement** for is possible for quasi-DC, strongly saturated magnets; **much smaller (but still significant) factor** when eddy currents and hysteresis play a major role

- Predicted optimal position based on DC FE up to **~30 mm off** $\Rightarrow$ **measurements necessary if high accuracy is needed**

- **Practical aspects** must also be considered when installing a probe: clearances, field level and gradients, external perturbations ….

- **Other possibilities** being explored:
  - explicit modelling of the magnetic length as a function of current, ramp rate, excitation history …
  - multiple sensors with constant coefficients

Thanks for your attention