IMMW21

International Magnetic Measurement Workshop

24th – 28th June 2019

Method for high-speed/resolution harmonics measurements with a multivertex rotating coil probe array

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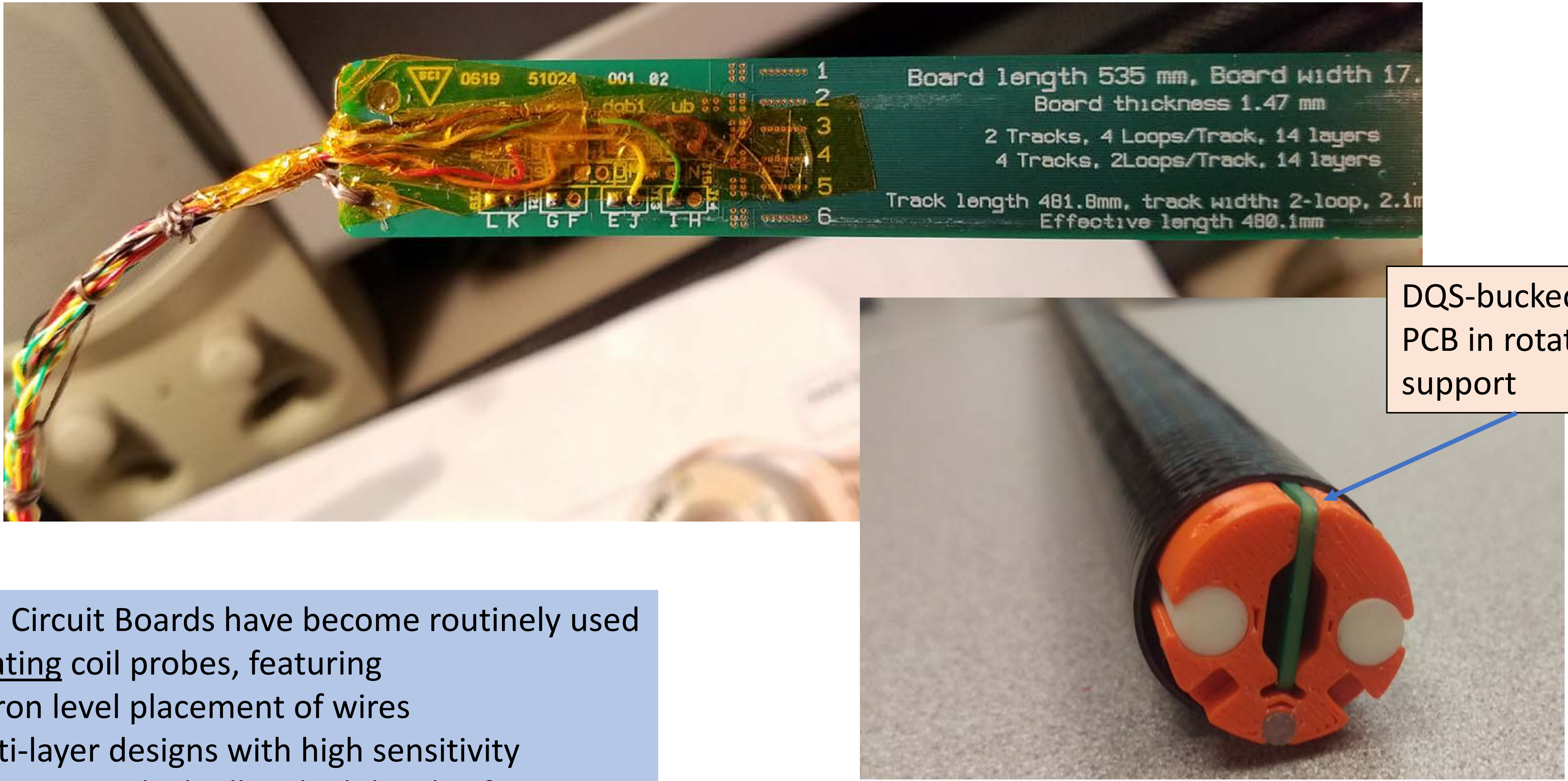
24Jun2019

ESRF

The European Synchrotron

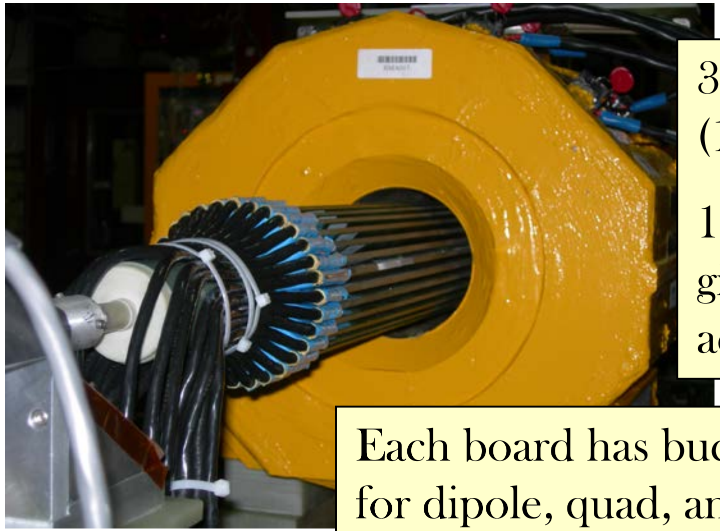


DQS-bucked PCB in rotating support

Printed Circuit Boards have become routinely used for <u>rotating</u> coil probes, featuring

- micron level placement of wires
- multi-layer designs with high sensitivity
- geometries which allow high levels of suppression of effects from main fields (“bucking”)

In the past have used a stationary array of PCB probes to make dynamic measurements on AC magnets for Fermilab Booster



32 Circuit Board Probes
(1.02m long, 22 layers each)

160 total channels (40 read for a given measurement, selected according to magnet type)

Sampling at 100kHz. Time resolution is 10 µsec

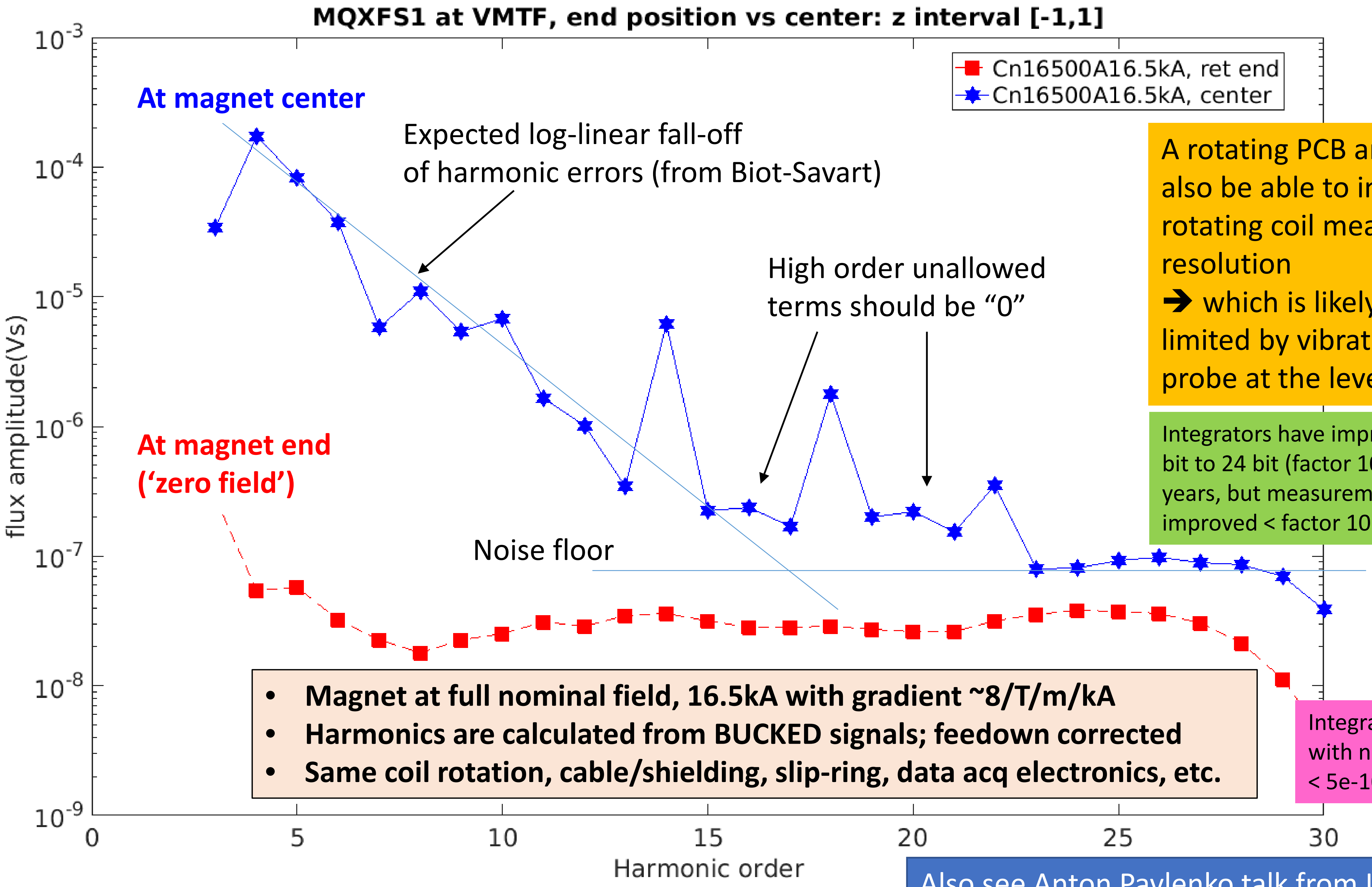
Each board has bucking for dipole, quad, and sextupole and low and high gain unbucked

This worked well, but was channel intensive and geared for high dB/dt current ramps. (Note that such a coil could be used as a quench antenna.)

Would like to **rotate** an array of PCB probes - this allows for fast measurements of magnetic field harmonic signature even in DC/slow-ramping conditions.

e.g., detailed magnetic measurements of eddy/coupling current effects, changes during injection porch 'decay'/snap-back, changes during mechanical events, etc.

J. DiMarco et al., "A Fast-Sampling, Fixed Coil Array for Measuring the AC Field of Fermilab Booster Corrector Magnets", IEEE Trans. on Applied Superconductivity, Vol. 18, No. 2, June 2008.



At magnet center

Expected log-linear fall-off of harmonic errors (from Biot-Savart)

High order unallowed terms should be "0"

At magnet end (‘zero field’)

Noise floor

- **Magnet at full nominal field, 16.5kA with gradient ~8/T/m/kA**
- **Harmonics are calculated from BUCKED signals; feedown corrected**
- **Same coil rotation, cable/shielding, slip-ring, data acq electronics, etc.**

A rotating PCB array might also be able to improve rotating coil measurement resolution

➔ which is likely still limited by vibrations of the probe at the level of ppm

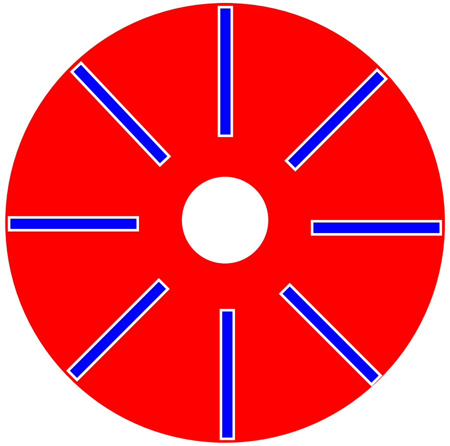
Integrators have improved from 14 bit to 24 bit (factor 1000) over 25 years, but measurement resolution improved < factor 10 (?)

Integrator noise with no-rotation is < 5e-10 Vs at n=30

Also see Anton Pavlenko talk from IMMW20

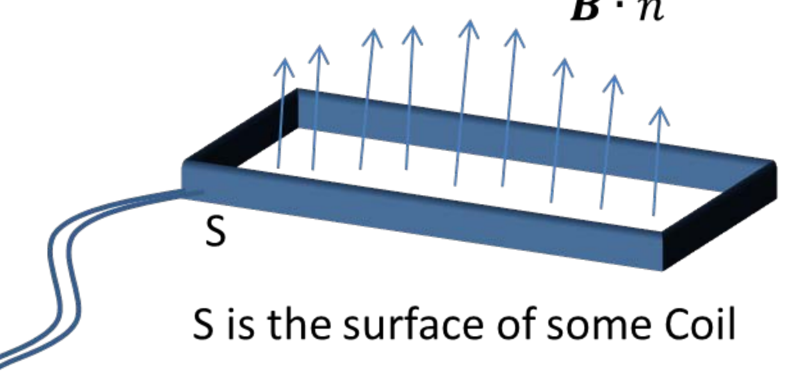
Investigate the possibility of achieving these two goals with a Multi-Vertex (MV) PCB array:

- high-speed measurements
- higher accuracy/resolution measurements
  (mitigate vibrations for all harmonic orders,
  reduce 1/f noise effects by measurements at faster speed, and use fact that integrators may no longer be limiting measurement resolution)

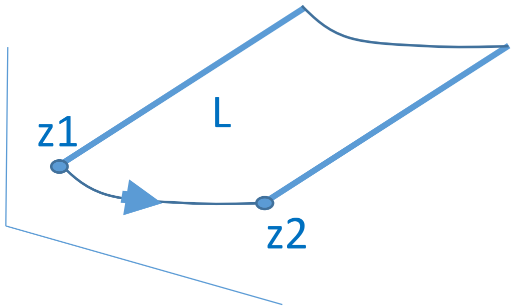


Throughout this presentation, an 8-vertex probe will be used as an example with 128 angular encoder positions. Scaling to a higher number of probe vertices and encoder angles is straightforward in the analysis.

Magnetic Field Determination



S is the surface of some Coil



Flux Field 'Arbitrary motion' end pos. start pos.

$$\Delta\boldsymbol{\Phi} = \mathbf{Re}\left[\frac{\boldsymbol{LR}}{\boldsymbol{n}}\sum_{\boldsymbol{n}=1}^{\infty}(\boldsymbol{B_n} + \boldsymbol{iA_n})\left(\left(\frac{\boldsymbol{z_2}}{\boldsymbol{R}}\right)^{\boldsymbol{n}} - \left(\frac{\boldsymbol{z_1}}{\boldsymbol{R}}\right)^{\boldsymbol{n}}\right)\right]$$

Measure flux *change* from integrated coil voltage during motion (or field change) ➔ **does not depend on path**, only start and stop of measurement.

From MEASURED ***ΔΦ***, along with **coil** & **motion information**, ➔ **B**

① Integrator or ADC/voltmeter

② Have to know the geometric parameters of the coil itself

③ path independent, but have to get the probe to move to correct, or at least known, motion positions

① Integrator or ADC/voltmeter

Using 24-bit integrator based on sigma-delta ADC (NI DSA modules and FPGA), or successive approximation based ADC (Fermilab developed integrator in compact RIO).

At least 8-channels of simultaneous acquisition are available.

New integrators for magnetic measurements

To keep number of channels low, planning to use only 1 signal from each PCB probe. Bucking the main fields is still very important in mitigating vibrations, so will adjust the bucking pattern on the probe so that there are UnBucked, DipoleBucked signals for the calibration (discussed next), and a signal which partially bucks dipole and quad at the level of 1/100 (which will be referred to as DQpBuck).

Since the DQpBuck would also have some sensitivity to dipole and quad fields, this can be used for main field strength determination (and center correction in the case of a quad field) as well as the harmonics.

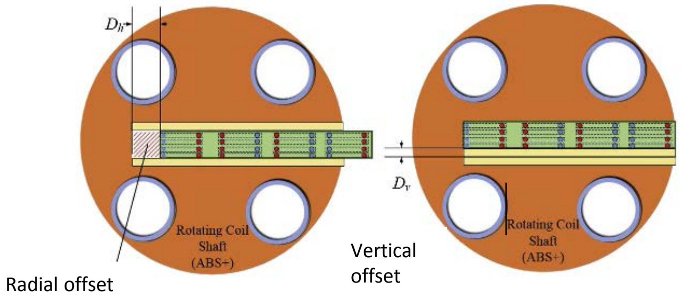
➔ Plan to use the UB, DB signals for calibration (see next slide), and then switch to DQpBuck signal from each of the 8 vertices for the MV acquisition

Signals used from each vertex will still take advantage of bucking main fields (at some level)

Calibration

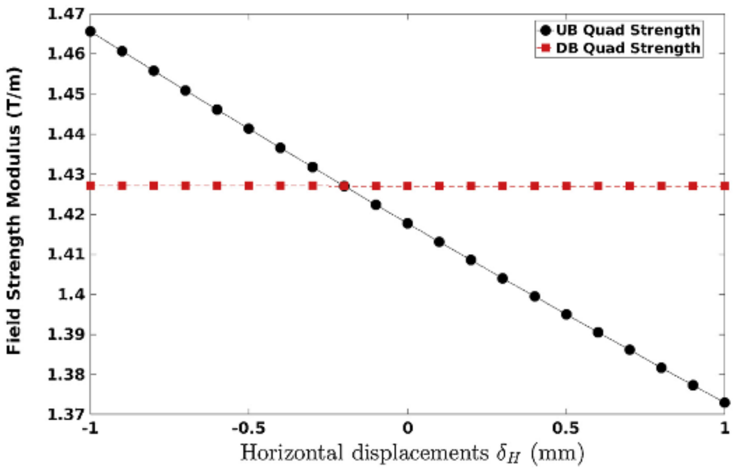
② Know the geometric parameters

PCB-probes allow for a straightforward, micron–level in-situ calibration in any quadrupole or higher field*.



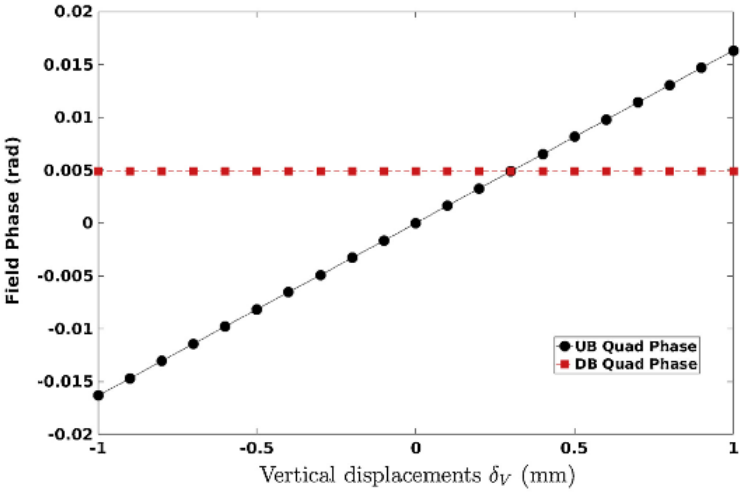
For MV probes, can apply this technique consecutively to pairs of PCBs to determine their radial and vertical offset positions (as per single PCB probes), as well as their relative angular offset. Any phase or gain variation among the main fields of the DQpBuck signals can also be determined

➔ This should lead to full knowledge of probe positions in MV array at level of microns.



$$D_h = \mathrm{Re}\left[\frac{F_2^{UB} * K_2^{DB} - F_2^{DB} * K_2^{UB}}{F_2^{DB} * Re\left(\frac{\partial K_2^{UB}}{\partial x}\right)}\right]$$

$$D_v = \mathrm{Im}\left\{\frac{F_2^{UB} * K_2^{DB} - F_2^{DB} * K_2^{UB}}{F_2^{DB} * Im\left(\frac{\partial K_2^{UB}}{\partial y}\right)}\right\}$$



* J. DiMarco et al., "Calibration technique for rotating PCB coil magnetic field sensors", Sensors and Actuators A: Physical, Volume 288, (2019), Pages 182-193

③

Correct, or at least known, motion positions

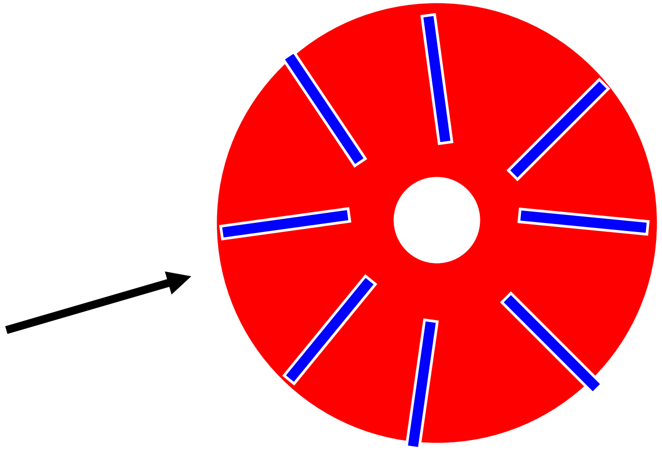
For now, assume that the probe rotates perfectly with no vibrations
Will return to this part of the problem later…

Assuming probe PCB positions well-known and that probe rotates ideally,
How can the multiple vertices be used?

1. **Average the results from 8 vertices -** Make a full rotation with the probe and just compare results among the 8 probes to see how big differences are – averages may cancel some of the effects of vibrations in addition to those removed by the partial bucking. (could also combine vertices for additional bucking, but won't discuss that here). Not high-speed, but may improve accuracy.
2. **'Speed multiplier' mode** - Can take 1/8 of the rotation samples from each probe to make a whole rotation (can do this for each 1/8 part of the rotation) – effectively this increases rotation speed by a factor eight. (and should consequently reduce influence of 1/f noise).
3. **'Snapshot' mode** (Nyquist limited) - Can measure quad and dipole (and in principle even 6p) for each encoder pulse (1kHz or more) as probe rotates. Straightforward to use this technique but higher order harmonics require increasingly more vertices: the challenges then lie in the multiplying number of cables, channels etc.

With calibrations, computing harmonics and averages for case 1) is straightforward.

For 2) and 3), where the signals are formed with contributions of all 8 probes, which generally are non-ideally located, the analysis is problematic



Standard rotating coil analysis

Expression for flux obtained from motion of coil from position Z1 to position Z2 (from slide 5)

$$\Delta\boldsymbol{\Phi} = \mathbf{Re}\left[\frac{\boldsymbol{LR}}{\boldsymbol{n}}\sum_{\boldsymbol{n=1}}^{\infty}(\boldsymbol{B_n}+\boldsymbol{iA_n})\left(\left(\frac{\boldsymbol{z_2}}{\boldsymbol{R}}\right)^{\boldsymbol{n}}-\left(\frac{\boldsymbol{z_1}}{\boldsymbol{R}}\right)^{\boldsymbol{n}}\right)\right]$$

instead of an arbitrary motion, suppose coil moves in a circular trajectory about the origin.

$$z_1 = |z_1|e^{i(\alpha_1)},\quad z_2 = |z_1|e^{i(\alpha_1+\Delta\theta)}$$

$$\boxed{\boldsymbol{\Phi(\theta)} = \mathbf{Re}\left[\sum_{\boldsymbol{n=1}}^{\infty}(\boldsymbol{B_n}+\boldsymbol{iA_n}) * \boldsymbol{K_n} * \boldsymbol{e^{in\theta}}\right]}$$

Field Expansion

Where TOTAL sensitivity is defined for the coil as the sum over ALL the wires:

$$\boldsymbol{K_n} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{N_{wires}}}\frac{\boldsymbol{L_jR}}{\boldsymbol{n}}\left(\frac{\boldsymbol{z_j^0}}{\boldsymbol{R}}\right)^{\boldsymbol{n}} * (-\mathbf{1})^{\boldsymbol{j}}$$

Note Kn does not depend on θ (θ moved into the φ expression)

Measurements produce a periodic waveform that can be Fourier analyzed. **Complex Fourier Coefficients**, F_n are determined such that

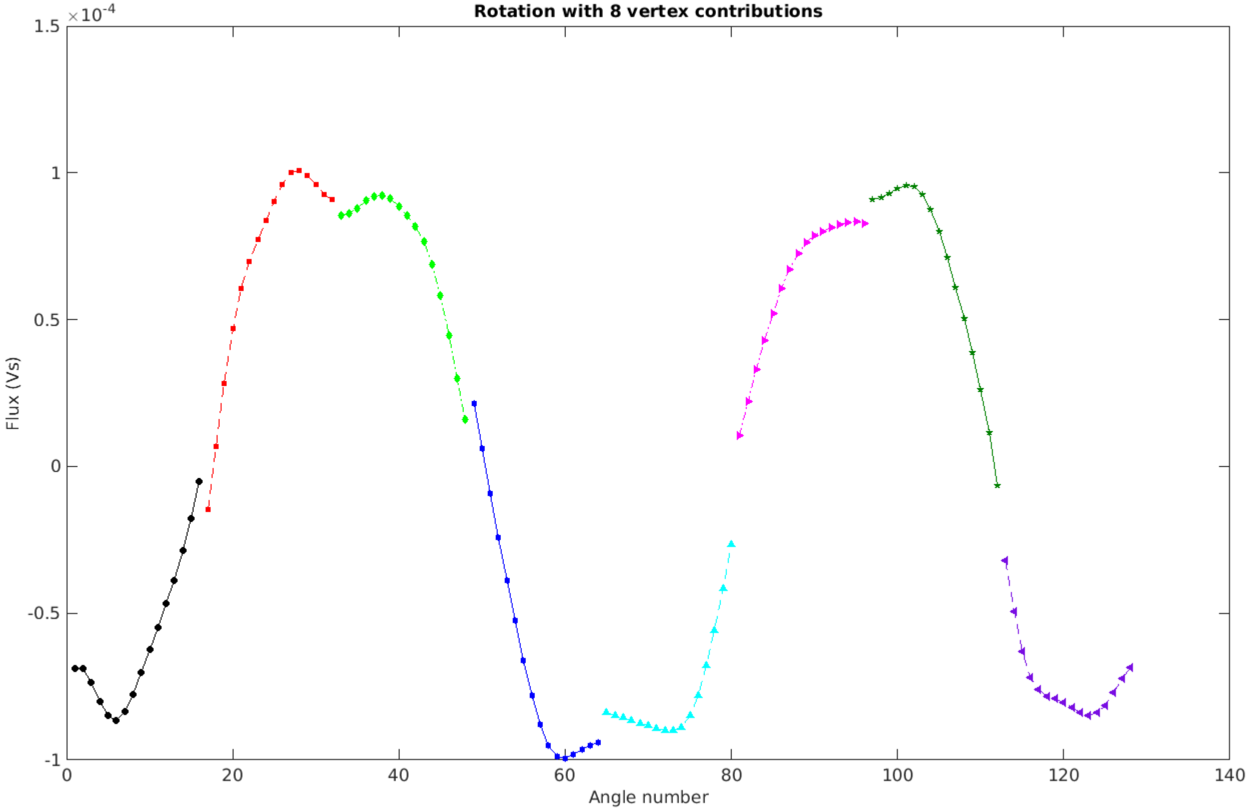
$$\Phi(\theta) = \sum_{n=1} F_n\, e^{in\theta}$$

F_n, can then be matched term by term with the Field Expansion. The **Field Coefficients** are determined from

$$\boldsymbol{C_n} = \boldsymbol{B_n} + \boldsymbol{iA_n} = \frac{\boldsymbol{F_n}}{\boldsymbol{K_n}}$$

Analyzing Multi-Vertex probe data

Standard FFT-based rotating coil analysis **will not work** ➔ The signal is comprised of data from the multiple vertex probes, and there are **discontinuities** because the K_n *changes for each vertex* (from vertex assembly (and probe vibration) errors).



Solution involves accounting for Kn being a function of θ, i.e. $K_n(\theta)$, and its changing depending on each vertex. FFT is not going to be amenable to this.

Alternative analysis - matrix solution to harmonics determination

Instead of using Fourier transform, write an expression for the angle dependent sensitivity, $K_n(\theta)$, such that the field expansion becomes

$$\boxed{\boldsymbol{\Phi(\theta) = \mathrm{Re}\left[\sum_{n=1}^{\infty}(B_n + iA_n) * K_n(\theta) * e^{in\theta}\right]}}$$

$$\boldsymbol{K_n(\theta) = \sum_{j=1}^{N_{wires}} \frac{L_j R}{n}\left(\frac{z_j(\theta)}{R}\right)^n * (-1)^j}$$

Sensitivity now can change for different probe vertices (and in general at each angle)

(Continued ➔)

‘Matsolve’ harmonics determination

To find harmonic coefficients, $\boldsymbol{C_n} = \boldsymbol{B_n} + \boldsymbol{iA_n}$, solve the harmonic expansion explicitly from the matrix equation:

Order, n ➔

Angle θ ⬇

$$\begin{pmatrix} \left(K_1(\theta_1)e^{i*1*\theta_1}\right) & \left(K_2(\theta_1)e^{i*2*\theta_1}\right) & \left(K_3(\theta_1)e^{i*3*\theta_1}\right) & & \left(K_{15}(\theta_1)e^{i*15*\theta_1}\right) \\ \left(K_1(\theta_2)e^{i*1*\theta_2}\right) & \left(K_2(\theta_2)e^{i*2*\theta_2}\right) & \left(K_3(\theta_2)e^{i*3*\theta_2}\right) & \cdots & \left(K_{15}(\theta_2)e^{i*15*\theta_2}\right) \\ \left(K_1(\theta_3)e^{i*1*\theta_3}\right) & \left(K_2(\theta_3)e^{i*2*\theta_3}\right) & \left(K_3(\theta_3)e^{i*3*\theta_3}\right) & & \left(K_{15}(\theta_3)e^{i*15*\theta_3}\right) \\ & \vdots & & \ddots & \vdots \\ & \vdots & & \vdots & \vdots \\ \left(K_1(\theta_{128})e^{i*1*\theta_3}\right) & \left(K_2(\theta_{128})e^{i*2*\theta_{128}}\right) & \left(K_3(\theta_{128})e^{i*3*\theta_{128}}\right) & & \left(K_{15}(\theta_{128})e^{i*15*\theta_{128}}\right) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ \vdots \\ C_{15} \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \vdots \\ \varphi_{128} \end{pmatrix}$$

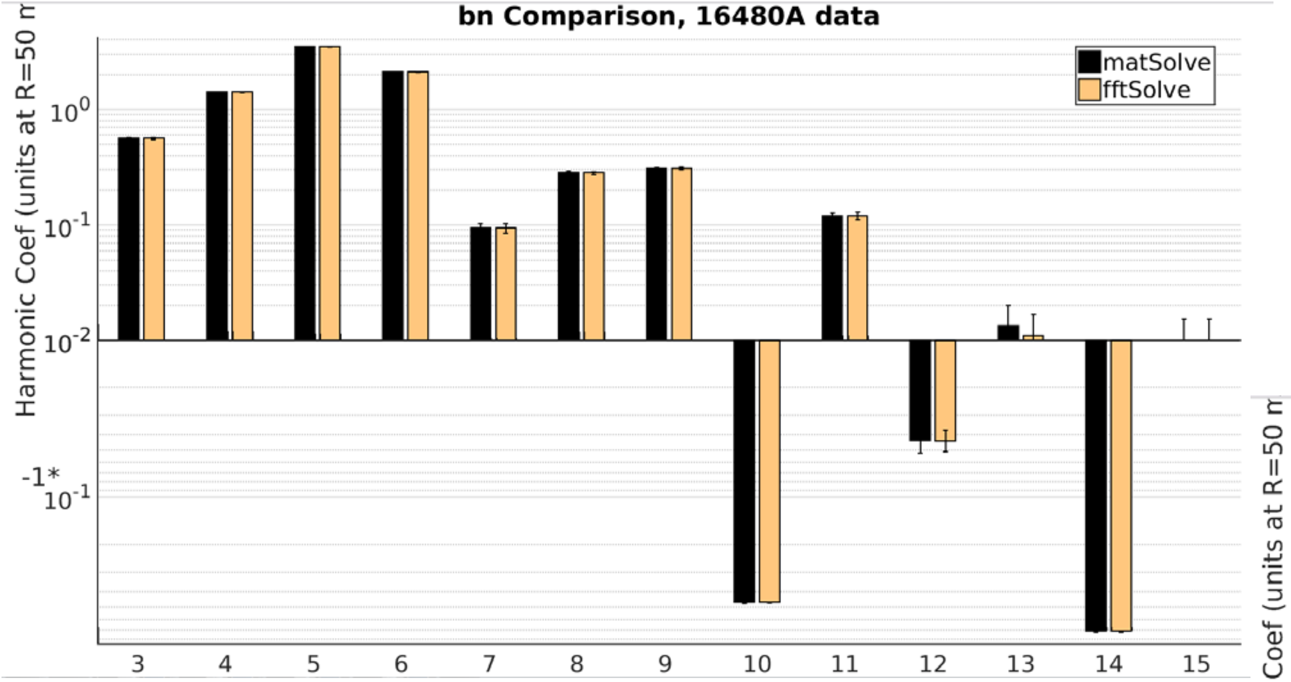
Different angles can have the sensitivity from different vertices

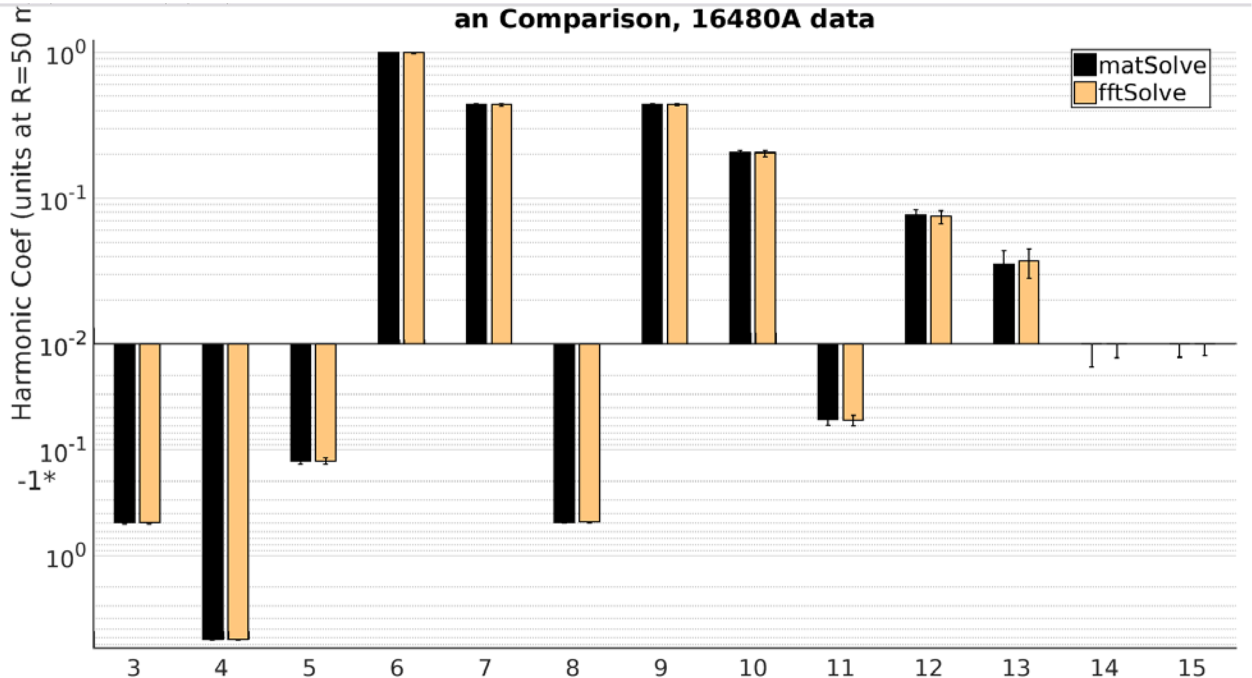
Over-determined – solving 15 (actually 30 because of Re, Im parts) quantities with 128 equations. This effectively functions as a least squares optimization of fit to harmonics.

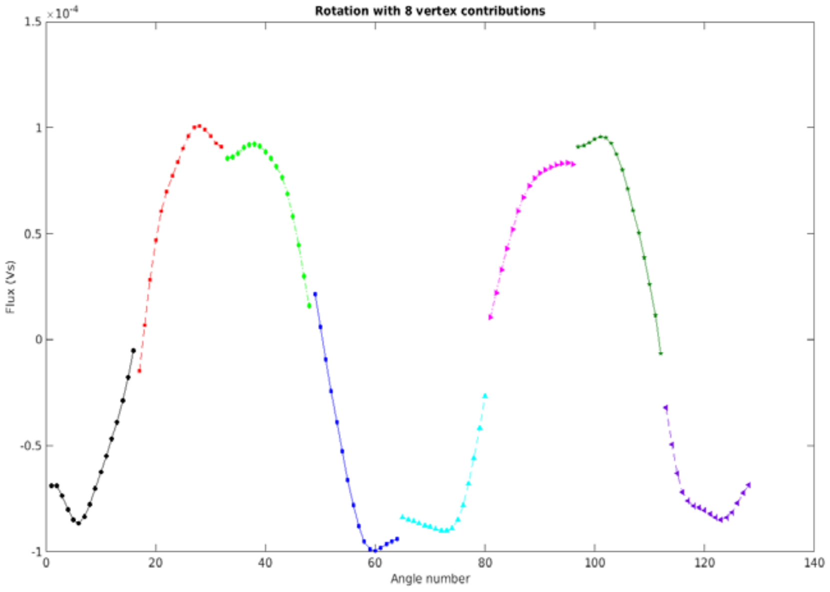
Speed of FFT goes as $N * \log_2 N$ (FFT effectively just an efficient algorithm for solving the above matrix in the normal case of regular theta dependence). The matsolve solution would take N^2 operations for all orders, but since solving only for n harmonics of interest, the computation is typically faster than the FFT.

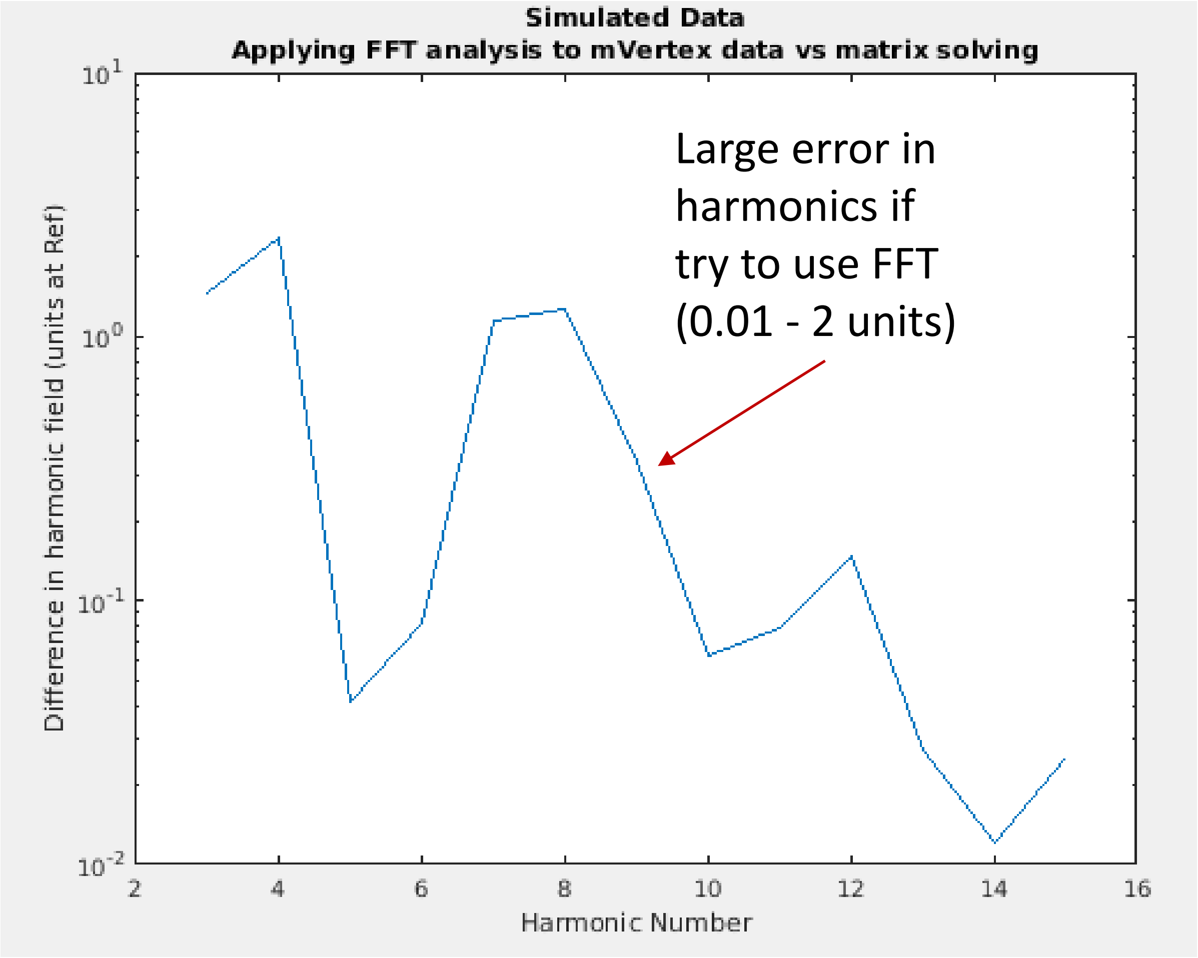
Compare 'Matsolve' harmonics vs standard FFT for actual magnet data

MQXFS1 data - All harmonic differences were less than 0.002 units between the two methods









Large error in harmonics if try to use FFT (0.01 - 2 units)

MatSolve analysis obtains the correct input harmonics from the simulated multi-vertex probe data shown earlier (error < 1e-4 units)

- Matrix solution for harmonics analysis works for simulated (high-speed) combined-vertex measurements.
- i.e. if max mechanical rotation of probe is 5Hz, now can achieve 40Hz sampling for all harmonics with 8 vertex probe.

➔ Have a method to work with analysis of multi-vertex probes

③ Correct, or at least known, motion positions

Mitigating effects of imperfect probe motion

Give us two handles to remove vibration effects – 1) bucking (for neighboring orders), and 2) motion determination and correction (all orders)

Now return to the issue of imperfect probe motion.

- First line of defense is bucking (suppressing) main fields. Spurious harmonics, ΔC_m , from vibration of amplitude D go as

$$\Delta C_m \sim \left(\frac{K_{n-1}}{K_m}\right)\left(\frac{D_{m-(n-1)}}{R_{ref}}\right) C_n$$

e.g. to generate fake sextupole (m=3) harmonics in a quadrupole field (n=2), the vibration amplitude should be of order 3-(2-1) = 2. So the expression becomes

$$\Delta C_3 \sim \left(\frac{K_1}{K_3}\right)\left(\frac{D_2}{R_{ref}}\right) C_2$$

Quadrupole amplitude is very large (10000 units)

To minimize spurious C_3 :

Need good rotation mechanics (in this case so 2nd order displacements D_2 are small compared to reference radius)

Dipole bucking - Suppress the sensitivity to dipole, K_1
(this effectively reduces displacement amplitude by the suppression ratio)

- Note that bucking does not really help the spurious harmonics generated from high order fields (e.g. spurious C7 from C6) – this limits resolution at high orders (since low order vibrations are more prevalent, only local neighboring harmonic fields are generally affected). One can also try to buck neighboring high-order fields (see A. Jain, IMMW 16)

In addition to bucking neighboring orders, will solve for the motion errors themselves (helps all orders) and account for their effects ➔ benefit from bucking AND from effectively correcting mechanical motion errors

③

Correct, or at least known, motion positions

Mitigating effects of imperfect probe motion (continued)

Determining actual probe motions -
Assume probe remains rigid, but moves during rotation

Can be displaced in X, Y and Theta, ➔ but affects all probe vertices simultaneously and by same amount

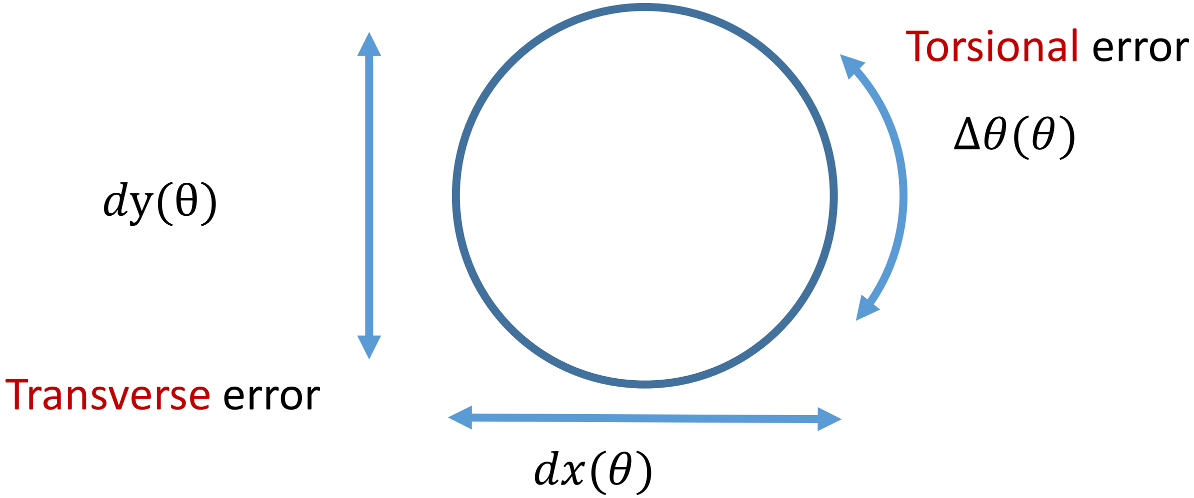
Since have interdependent data, can set up a system of equations to determine offsets at each angle.
Use methodology for Kn which have theta dependence $K_n(\theta)$

The harmonic error is caused by the $K_n(\theta)$ variations, so can analyze the coupled equations and determine the displacements.

Once the displacements have been determined, can find new $K_n(\theta)$, which take into account the displacements, and analyze using the matrix solving technique

Determining actual probe motion

Errors that cause flux to be different than perfect motion are caused by the change in winding sensitivity stemming from vibrations



Determining actual probe motion (continued)

For each angle, $\theta_1, \theta_2, \dots, \ \theta_{128}$, express the measured flux (e.g. for θ_1) as

Flux contribution terms →

↓ Vertices

Unknown errors at θ_1

Flux at each vertex at θ_1

$$\begin{pmatrix} K_n^{V_1 nom}(\theta_1)\cdot C_n & \frac{\partial K_n^{V_1 nom}(\theta_1)}{\partial x}\cdot C_n & \frac{\partial K_n^{V_1 nom}(\theta_1)}{\partial y}\cdot C_n & \frac{\partial K_n^{V_1 nom}(\theta_1)}{\partial \theta}\cdot C_n \\ K_n^{V_2 nom}(\theta_1)\cdot C_n & \frac{\partial K_n^{V_2 nom}(\theta_1)}{\partial x}\cdot C_n & \frac{\partial K_n^{V_2 nom}(\theta_1)}{\partial y}\cdot C_n & \frac{\partial K_n^{V_2 nom}(\theta_1)}{\partial \theta}\cdot C_n \\ \vdots & & \ddots & \vdots \\ \vdots & & & \vdots \\ K_n^{V_8 nom}(\theta_1)\cdot C_n & \frac{\partial K_n^{V_8 nom}(\theta_1)}{\partial x}\cdot C_n & \frac{\partial K_n^{V_8 nom}(\theta_1)}{\partial y}\cdot C_n & \frac{\partial K_n^{V_8 nom}(\theta_1)}{\partial \theta}\cdot C_n \end{pmatrix} \begin{pmatrix} 1 \\ dx(\theta_1) \\ dy(\theta_1) \\ d\theta(\theta_1) \end{pmatrix} = \begin{pmatrix} \varphi^{V_1}(\theta_1) \\ \varphi^{V_2}(\theta_1) \\ \vdots \\ \vdots \\ \varphi^{V_8}(\theta_1) \end{pmatrix}$$

Contributions from sensitivity derivatives

Solve for these

Where the $\frac{\partial K_n^{V_1 nom}(\theta_1)}{\partial x}$ e.g. are the sensitivity derivatives for a particular vertex at a particular angle and are calculated from the difference of $K_n^{V_1 nom}(\theta_1)$ found at nominal coordinate positions and with all x-coordinates shifted by a small amount

The C_n are determined from the nominal vertices analysis – when the analysis is repeated including the vibrations, these can be iterated if necessary

Determining actual probe motion (continued)

For each angle, obtain the vibration offsets

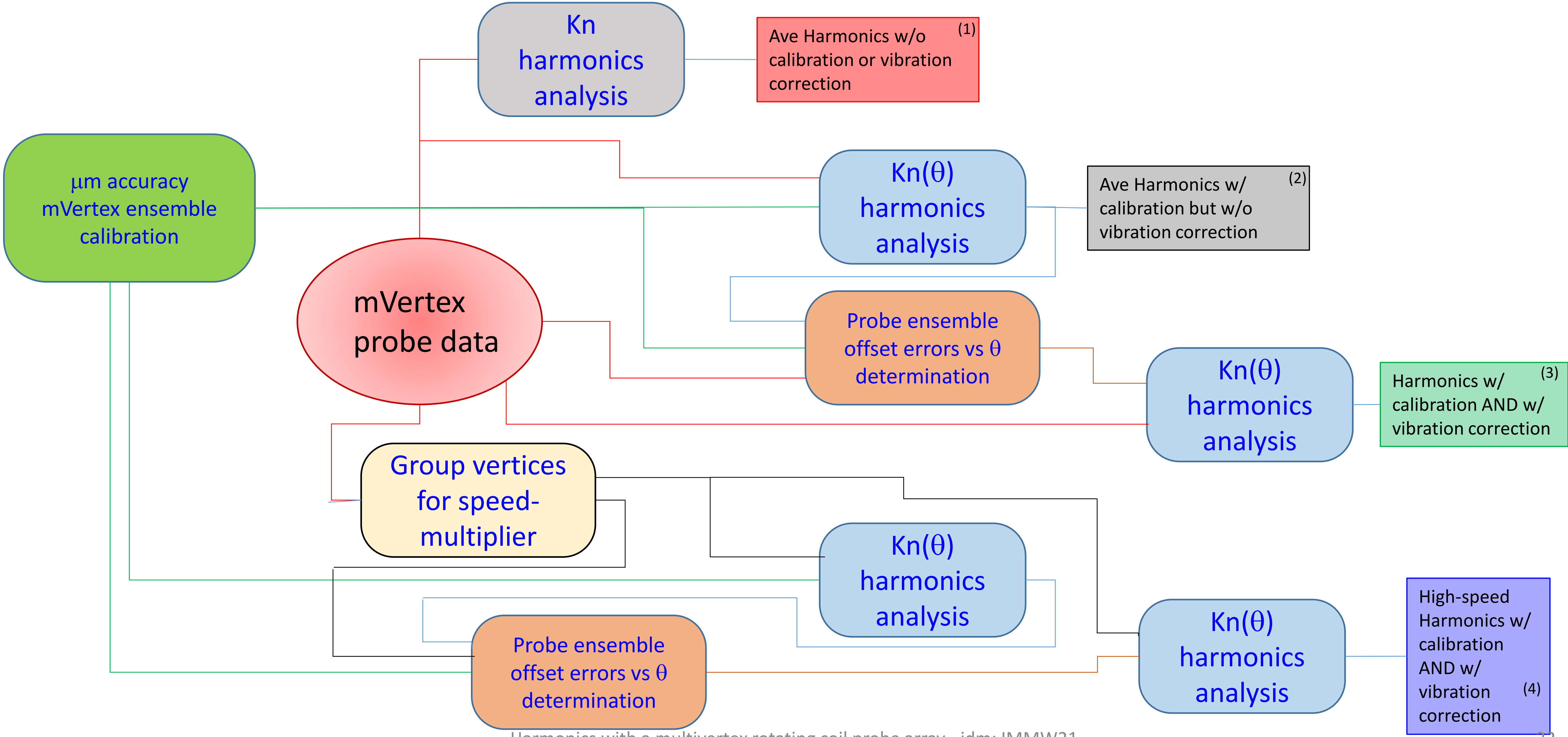
$$\begin{pmatrix} dx(\theta_1), dy(\theta_1), d\theta(\theta_1) \\ dx(\theta_2), dy(\theta_2), d\theta(\theta_2) \\ \vdots \\ dx(\theta_{128}), dy(\theta_{128}), d\theta(\theta_{128}) \end{pmatrix}$$

Then go back and determine $\boldsymbol{K_n(\theta)}$ for each vertex based on its original calibration plus the vibration offsets determined above (noted here by $\boldsymbol{z_j^{vib}}$)

$$\widetilde{\boldsymbol{K_n}}\,(\boldsymbol{\theta}) = \sum_{j=1}^{N_{wires}} \frac{\boldsymbol{L_j R}}{\boldsymbol{n}} \left(\frac{\boldsymbol{z_j(\theta)} + \boldsymbol{z_j^{vib}(\theta)}}{\boldsymbol{R}} \right)^{\boldsymbol{n}} * (-\boldsymbol{1})^{\boldsymbol{j}}$$

Can then use the matsolve technique with these $\widetilde{\boldsymbol{K_n}}$ to determine harmonics from multiple vertices (effectively multiplying rotation speed) and being fully compensated for vibration errors.

Flow chart of analysis results (simulated data)



μm accuracy mVertex ensemble calibration

mVertex probe data

Kn harmonics analysis

Ave Harmonics w/o calibration or vibration correction (1)

Kn(θ) harmonics analysis

Ave Harmonics w/ calibration but w/o vibration correction (2)

Probe ensemble offset errors vs θ determination

Kn(θ) harmonics analysis

Harmonics w/ calibration AND w/ vibration correction (3)

Group vertices for speed-multiplier

Kn(θ) harmonics analysis

Probe ensemble offset errors vs θ determination

Kn(θ) harmonics analysis

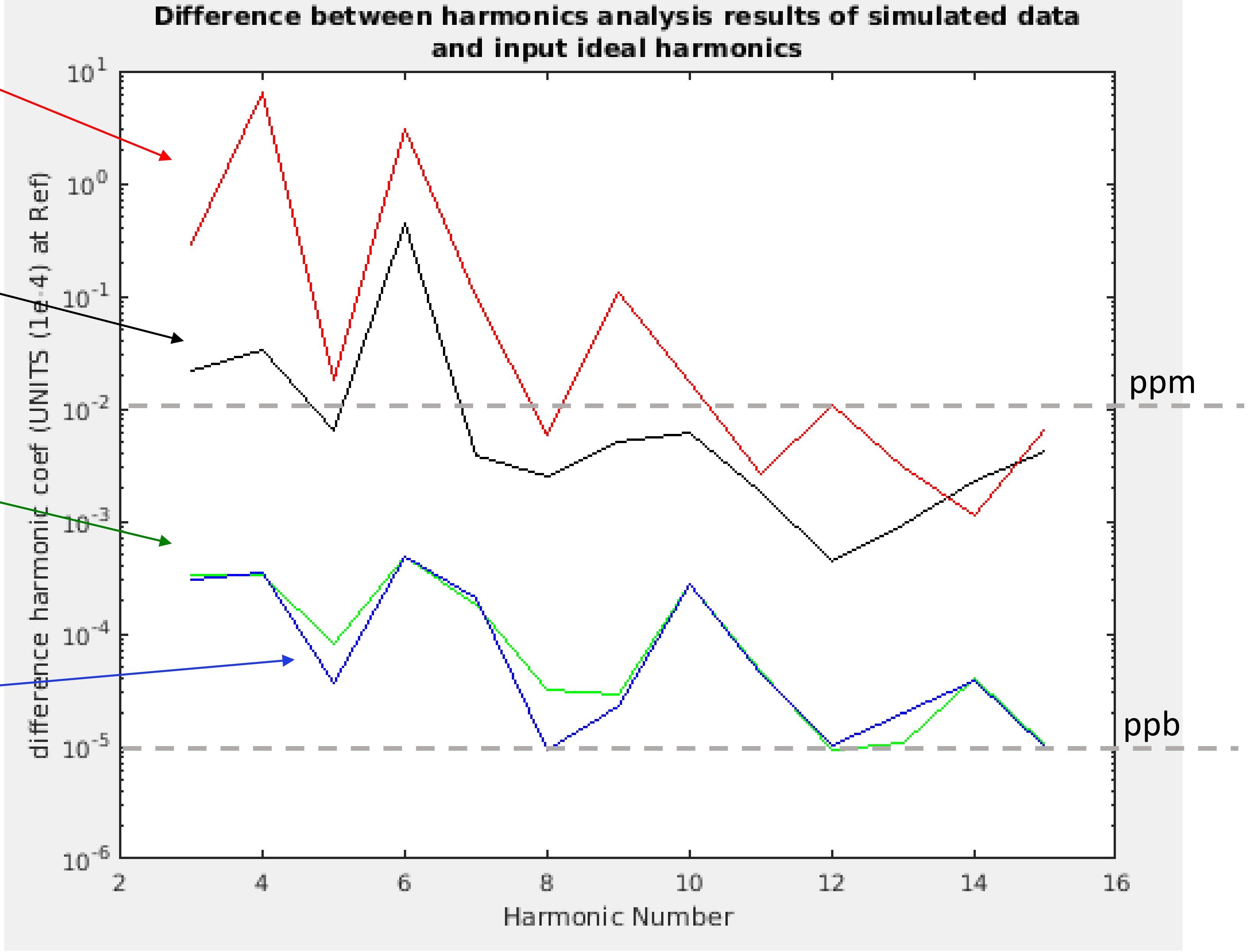
High-speed Harmonics w/ calibration AND w/ vibration correction (4)

Average of 8 vertices results without compensating PCB placement errors or probe vibration

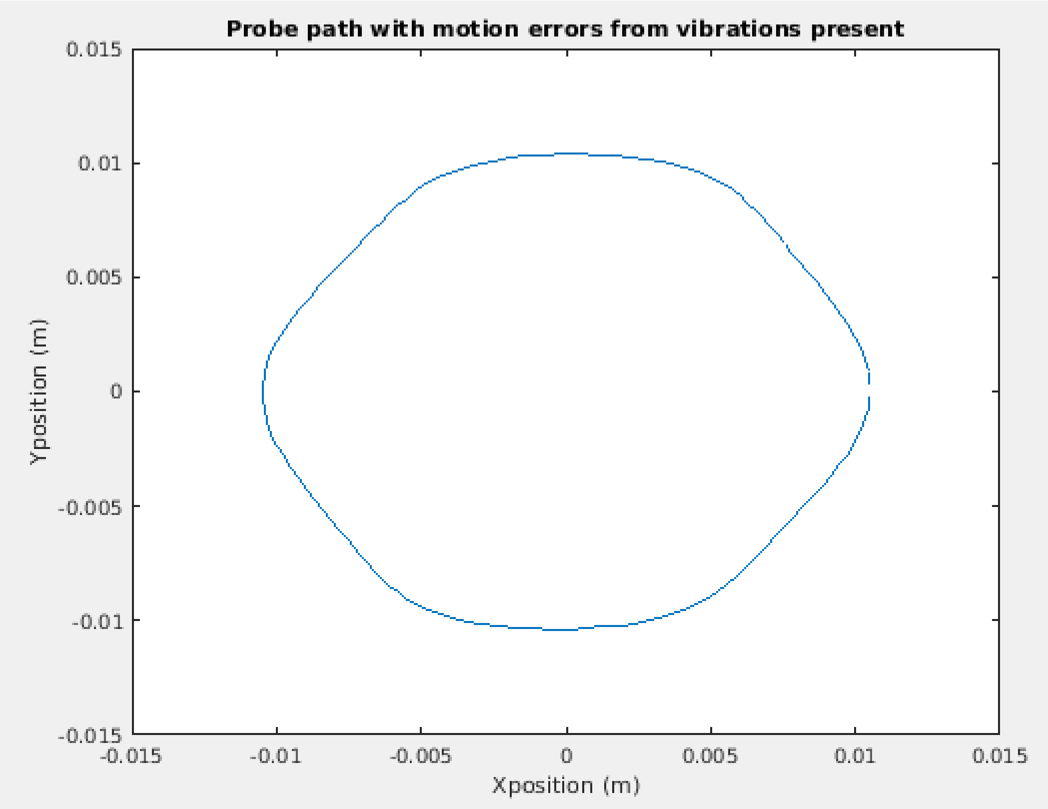
Average of 8 vertices results <u>with compensation</u> for PCB placement errors but no probe vibration correction

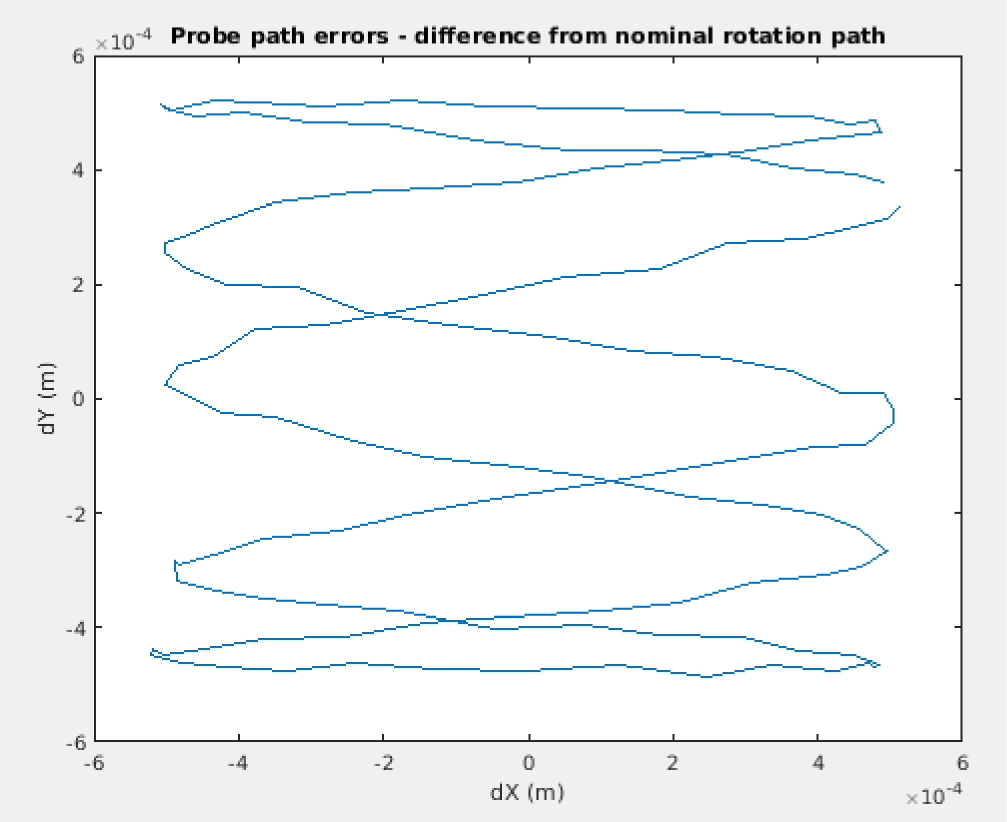
Average of 8 vertices results <u>with compensation</u> for PCB placement errors <u>and probe vibration correction</u>

Speed multiplier result with full analysis



Probe motion determined from analysis on simulated data





Motion errors on the order of 0.5mm

Magnetic Field Determination with MV probes

From MEASURED **$\Delta\Phi$**, along with **coil** & **motion information**, ➔ **B**

Integrator or ADC/voltmeter

Have to know the geometric parameters

path independent, but have to get the probe to move to correct, or at least known, motion positions

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8 (or more) channel integrator reading partially bucked windings

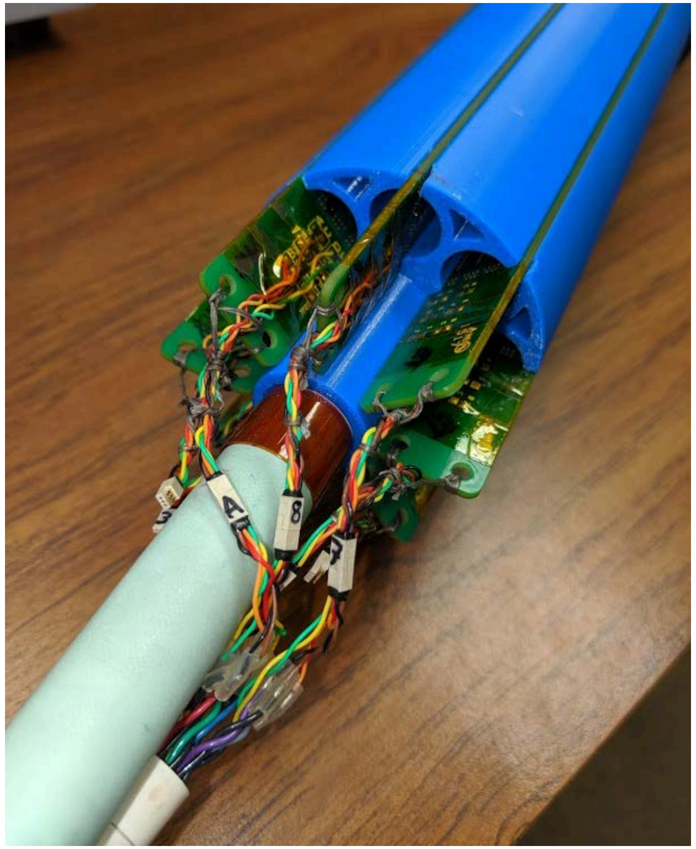
PCB calibration technique

Multi-vertex determination of probe motion with theta-dependent sensitivity (Kn(θ)), and Kn(θ) derivatives

Matrix based harmonic calculation technique

Summary and status

The localization of probe windings afforded by a high-accuracy PCB probe calibration technique enables use of multi-vertex arrangement of probes; which in turn enables the possibility of measuring at higher effective rotation speeds and compensating for motion errors (which may also lead to higher accuracy/resolution measurements).



Simulation and analysis software have been developed.

A multi-vertex test coil is assembled and ready for data taking to explore what might be achievable with these techniques.

Thanks for your attention!